Math 762 Homework Assignment, Due Thursday, April 19

1. Prove the following:

a. Let X be the convex hull of a finite set of points, $X =< p_1, p_2, \ldots, p_n >\subset \mathbb{E}^d$, and let p be in the relative interior of X. Show that p is in the relative interior of at most $2d$ of the points $p_1, p_2, \ldots, p_n$. (This is a theorem of Steinitz.)

b. Let $H_1, H_2, \ldots, H_n$ be closed half spaces containing the origin in $\mathbb{E}^d$ such that $\cap_i H_i = \{0\}$. Show that there is a set of at most $2d$ of the half spaces whose intersection is exactly the origin.

c. Let $G(p)$ be a tensegrity framework with n vertices, $m > 0$ cables and struts and b bars, such that it is infinitesimally rigid in $\mathbb{E}^d$, and the subframework consisting of just the bars has only the 0 equilibrium stress. Furthermore, suppose that $G(p)$ is minimal in the sense that $G(p)$ is not infinitesimally rigid if any cable or strut is removed. Show that $nd - d(d+1)/2 - b + 1 \leq m \leq 2(nd - d(d+1)/2 - b)$.

c. Estimate the minimum number of cables that are needed to add to the edges (bars) of the cube in Euclidean 4-space so that the resulting tensegrity is infinitesimally rigid in 4-space? Extra credit for anyone that can find the proveably minimum number of bars.

2. Suppose that you have k rigid bodies in $\mathbb{E}^d$, each of which whose vertices have affine span all of $\mathbb{E}^d$. Show that you need to add at least $d(d+1)(k-1)/2$ bars in order to make them all infinitesimally rigid in $\mathbb{E}^d$.

3. A bar framework $G(p)$ in $\mathbb{E}^d$ is called infinitesimally redundantly rigid if it is infinitesimally rigid in $\mathbb{E}^d$ and it remains infinitesimally rigid after the removal of any bar. Show that $G(p)$ is infinitesimally redundantly rigid if is infinitesimally rigid and there is an equilibrium stress that is non-zero on all of is members.

4. Find a combinatorial characterization of redundant rigidity for a generic realization of a bar graph in the plane. You may assume that the graph is minimally redundantly rigid in the sense that it is not redundantly rigid after the removal of any bar.