Math 762 Homework Assignment, Due Thursday, April 26

1.

a. Let $p_1 = (x_1, y_1), p_2 = (x_2, y_2)$ be two distinct non-zero points in the plane. Let $p_3 = (-x_1, y_1), p_5 = (x_1, -y_1), p_7 = (-x_1, -y_1)$ and $p_4 = (-x_2, y_2), p_6 = (x_2, -y_2), p_8 = (-x_2, -y_2)$. Construct the bar framework where there are bars between each vertex with an even coordinate and a n odd coordinate. This is $G(p)$ where G is the complete bipartite graph $K(4, 4)$. Show this $G(p)$ is a mechanism. In other words $G(p)$ is not rigid in the plane.

b. Show that $K(4, 4)$ is generically rigid in the plane.

2. Let G be $K(3, 3)$, the complete bipartite bar graph between two sets of three vertices.

a. Consider a configuration of vertices for G such that one set of three vertices lies on the x-axis in the plane, but none at the origin. Similarly, assume that the other set of three vertices is on the y-axis, none at the origin. Show that this bar framework is a mechanism in the plane.

b. Suppose that a framework with the edge lengths of part a.) is in three-space. Show that one of the sets of three vertices are collinear.

3. Suppose that $G(p)$ is a bar framework in $\mathbb{E}^d$ that is *isostatic*, which means that it is infinitesimally rigid, but it has only the 0 equilibrium stress. Consider the rigidity map $f : \mathbb{E}^{nd} \to \mathbb{E}^e$, where n is the number of vertices of G and e is the number of bars of G.

a. Show that near the image of the configuration p, f is onto. Use that to show that if any of the bars of G are removed, the resulting framework is a mechanism.

b. Suppose that P is a convex polytope in three-space such that each face is a triangle. Construct the bar framework $G(p)$ with the vertices of P as vertices and the edges of P as bars. Show that if any bar is removed from G the resulting framework is a mechanism.