1. We say that a sequence of points $x_1, x_2, \dots \in X$ **converges to** $x \in X$ if for every neighborhood U of x, there is an integer $n > 0$ so that $x_i \in U$ for every $i \geq n$.
   Show that if $f : X \to Y$ is a continuous function and if $\{x_i\}$ is a sequence that converges to x, then $\{f(x_i)\}$ converges to $f(x)$.
2. Show that a function $f : X \to Y$ between two topological spaces is continuous if and only if $f^{-1}(A)$ is closed for every closed subset $A \subseteq Y$.
3. Let X be a topological space. We say that $x \sim y$ if there is a path in X from x to y.
   (a) Show that this is an equivalence relation on X.
   (b) What are the equivalence classes for this relation for $X =$ the topologist's sine curve?
4. Let X be an infinite set with the **finite complement topology**. That is, a set $A \subseteq X$ is open if its complement is a **finite set** or all of X. Show that X with this topology is connected.
5. Suppose that $A \subseteq X$. The **boundary** of A, denoted $\mathrm{Bd}(A)$, is the set $\overline{A} \cap \overline{(X - A)}$.
   (a) If A is connected, are the interior $\mathrm{int}(A)$ and boundary $\mathrm{Bd}(A)$ also connected? Prove or find a counterexample.
   (b) If the interior $\mathrm{int}(A)$ and the boundary $\mathrm{Bd}(A)$ are connected, then is A necessarily connected? Prove or find a counterexample.
6. Show that if A is connected, $\overline{A}$ is connected.
7. Let D_1 and D_2 be two disjoint open discs in $\mathbb{R}^2$ that are tangent to each other, as shown in the figure below.

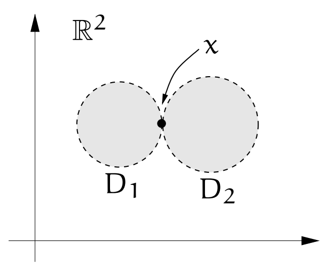

FIGURE 1. A pair of open tangent discs D_1 and D_2 in $\mathbb{R}^2$, with point of tangency x.

   Which of the following subsets of $\mathbb{R}^2$ are connected? Why or why not?
   (a) $D_1 \cup D_2$
   (b) $\overline{D}_1 \cup \overline{D}_2$
   (c) $\overline{D}_1 \cup D_2$
8. Consider the capital letters of the alphabet, as below, in the sans serif style with no adornments.

   ABCDEFGHIJKLMNOPQRSTUVWXYZ

   Each letter is a topological space, with the subspace topology inherited from R^2.
   (a) Prove that K is not homeomorphic to X.
   (b) Give an explicit homeomorphism from O to D
   (c) Consider the equivalence relation "is homeomorphic to" on the set of these letters. What are its equivalence classes?
9. (EXTRA CREDIT) Let X be a subspace of $\mathbb{R}^n$ with the standard topology, for some n. Suppose $X = A \sqcup B \sqcup C$ is a disjoint union of path-connected subsets.
   (a) Show that if $A \sqcup B$, $B \sqcup C$ and $A \sqcup C$ are **not** connected, then X cannot be connected.
   (b) Find an example of such X, A, B, and C all path-connected so that **none** of $A \sqcup B$, $B \sqcup C$ and $A \sqcup C$ is path-connected.