1. Let X be a topological space. Prove that "is path homotopic to" is an equivalence relation on the set of (continuous) paths $f : I \to X$.
2. Given spaces X and Y, let $[X, Y]$ denote the set of homotopy classes of continuous maps $f : X \to Y$.
   a. Show that the set $[X, I]$ has a single element.
   b. Show that if X is path connected, the set $[I, X]$ consists of a single element.
3. A space X is contractible if the identity map $i_X : X \to X$ is **nullhomotopic**: that is, homotopic to a constant map.
   a. Show that I is contractible.
   b. Show that a contractible spaces is path connected.
   c. Show that if Y is contractible, then for any space X, the set $[X, Y]$ consists of a single element.
4. Show that a space X is contractible if and only if every map $f : X \to Y$, for an arbitrary space Y, is nullhomotopic.
5. Recall that a **retract** from X to a subset A is a continuous map $r : X \to A$ such that $r(a) = a$ for every $a \in A$. Show that a retract of a contractible space is contractible.
6. A **deformation retraction** from X to A is a continuous map $F : X \times I \to X$ so that $F(x, 0) = x$ and $F(x, 1) \in A$ for every $x \in X$, and $F(a, t) = a$ for every $a \in A$ and $t \in I$. (This is stronger than requiring that the retraction $F(\cdot, 1)$ is homotopic to the identity.) Show that if there is a deformation retraction from X to A, then X is homotopy equivalent to A.
7. Show that if a space X deformation retracts to a point $x \in X$, then for each neighborhood U of $x \in X$, there exists a neighborhood $V \subset U$ of x such that the inclusion $V \hookrightarrow U$ is nullhomotopic.
8. Suppose that X is path connected. We say that X is simply connected if $\pi_1(X, x_0)$ is the trivial (one-element) group for some basepoint $x_0 \in X$ (and hence for every basepoint $x \in X$). Show that X is simply connected if and only if there is a unique homotopy class of paths connecting any two points in X.
9. (EXTRA CREDIT) Let $(G, \cdot)$ be a path connected topological group, with identity element $e \in G$. Show that the fundamental group $\pi_1(G, e)$ is an abelian group. That is, show that for any two loops $f, g : I \to G$, the paths $f \star g$ and $g \star f$ are path homotopic.

   HINT: Consider the function $f \cdot g : I \times I \to G$ defined by $f \cdot g(s, t) = f(s) \cdot g(t)$.