All section numbers (*e.g.* §1.1) refer to sections in the textbook *Elementary Differential Geometry*, by Andrew Pressley. At the top of you solutions, please indicate **how much time** you spent working on this problem set.

1. Please do 3 problems of your choice from §1.1, problems 1.1–1.10.
2. Express the *folium of Descartes*
$$\mathcal{F} = \{(x, y) \in \mathbb{R}^2 \mid x^3 + y^3 = 3xy\}$$
as a parametrized curve, using as parameter the slope t of a line passing between the origin and the point (x, y). Use your parametrization to draw the curve.
3. Find the position and tangent vector of the helix
$$\gamma(t) = (\cos(t), \sin(t), t)$$
at times $t = 0$ and $t = \pi/2$.
4. Prove that if we dilate Euclidean space by a factor of c (*i.e.* move each point x to the point $c \cdot x$), where $c \in \mathbb{R}$ is a positive constant, then we dilate lengths by the same factor.
5. Prove that the lengths of curves are unchanged by any transformation of $\mathbb{R}^n$ that preserves lengths of all line segments.
6. Consider the graph of the function
$$y = x \sin\left(\frac{\pi}{x}\right),$$
for $0 < x < \pi$. Find the value of y at the points $x = \frac{1}{n}$, $x = \frac{1}{n+\frac{1}{2}}$, and $x = \frac{1}{n+1}$.
Prove that the piece of the curve from $x = \frac{1}{n+1}$ to $x = \frac{1}{n}$ has arc-length at least
$$\frac{2}{n + \frac{1}{2}}.$$
Use this to show that this curve has infinite arc-length. Sketch the curve.
7. Please do problem 1.14 from §1.3.
8. Find the arc-length function s of the curve given in polar coordinates by
$$r = \frac{\theta^2}{4}.$$
Find the length of the piece of the curve parametrized by $-\pi \leq \theta \leq \pi$. Draw that piece.
9. Prove that the curve $y^2 = x^2(x + 1)$ (the **nodal cubic**) is parametrized by
$$\gamma = (t^2 - 1, t(t^2 - 1)),$$
with every point (x, y) of the nodal cubic reached for some value of t. (HINT: try taking t to be the slope of the line between that point and the origin. Show that using this t value for the parameter, the curve reaches the point (x, y).) Draw the curve.