

All section numbers (e.g. §1.1) refer to sections in the textbook *Elementary Differential Geometry*, by Andrew Pressley. At the top of your solutions, please indicate **how much time** you spent working on this problem set.

1. Please do problem 2.2 in §2.1.
2. Compute the curvature for the following curves.

(a) $\gamma(t) = \left(\frac{1}{\sqrt{2}} \cos(t), \frac{1}{\sqrt{2}} \cos(t), \sin(t) \right)$

(b) $\gamma(t) = (\cosh(t), \sinh(t), t)$

(c) $\gamma(t) = \left(t, \frac{t^2}{2}, \frac{t^3}{3} \right)$

(d) $\gamma(t) = (e^t \cos(t), e^t \sin(t), e^t)$

3. Please do problems 2.4 and 2.8 in §2.2.

4. Prove that the function

$$f(t) = \begin{cases} e^{-\frac{1}{t^2}} & \text{if } t > 0, \\ 0 & \text{if } t \leq 0, \end{cases}$$

is smooth. Be careful about what happens as $t \rightarrow 0$! Prove that the curve

$$\gamma(t) = \begin{cases} (e^{-\frac{1}{t^2}}, 0) & \text{if } t < 0, \\ (0, e^{-\frac{1}{t^2}}) & \text{if } t > 0, \\ (0, 0) & \text{if } t = 0, \end{cases}$$

is smooth. Prove that it is regular when $t \neq 0$. Draw it and find its curvature.

5. Prove that the curvature of a plane curve $y = f(x)$ is given by

$$\kappa = \frac{|f''|}{(1 + (f')^2)^{\frac{3}{2}}}.$$

6. (a) Which point on the graph of $y = \ln(x)$ has the largest curvature? First make a guess from a decent drawing of the graph, then compute the answer.
(b) Answer the same question for the graph of $y = e^x$.
(HINT: after doing (a), this should take about 10 seconds.)