This problem set should be a little shorter – I am just trying to make sure you are keeping up with what's happening in lecture. Please write up solutions to the following problems.

1. Is the set of points (x, y, z) for which (x, y) lies in the limaçon a smooth surface? Why or why not?
2. If we rotate a curve in the plane to make a surface of revolution, for which curves is it a smooth surface?
3. An interesting parametrization of the sphere is given by **stereographic projection**. We parametrize the sphere minus the north pole $(0, 0, 1)$ by the xy-plane assigning to each (u, v) the point where the line through $(0, 0, 1)$ and $(u, v, 0)$ intersects the sphere. Draw a picture of this parametrization, and prove that it can be expressed as
$$\sigma(u, v) = \left(\frac{2u}{u^2 + v^2 + 1}, \frac{2v}{u^2 + v^2 + 1}, \frac{u^2 + v^2 - 1}{u^2 + v^2 + 1}\right).$$
Show that this leads to an atlas for the sphere that consists of two charts.
4. Prove that a point in $\mathbb{R}^3$ is not a surface. Prove that a line in $\mathbb{R}^3$ is also not a surface.
5. Consider the generalized cylinder on the graph of
$$y = \sin\left(\frac{1}{x}\right),$$
for $x \neq 0$. Is this a smooth regular surface?

Please think about the following problems from the textbook. All section numbers (*e.g.* §1.1) refer to sections in the textbook *Elementary Differential Geometry*, by Andrew Pressley. You do not need to write up solutions for these problems, but you should certainly attempt them.

6. Problems 4.3 and 4.4 in §4.1.
7. Problems 4.6 and 4.7 in §4.2.
8. Problems 4.13 and 4.14 in §4.3.
9. Problems 4.18 and 4.19 in §4.4.