Math 304
Homework #1 Solutions

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Solow 1.1 a, c, e, and f are statements. In each case, translating the symbols into words produces a *sentence*, whereas translating the symbols into words in both b and d produces merely a noun phrase.

Solow 1.2 a) This is true. The cube root of any integer n and is equal to the unique real number r such that $r^3 = n$.

Some people interpreted the statement as meaning "For any integer n, *all* cube roots (in the complex numbers) are real," which is a defensible interpretation as well. Under this interpretation, the statement is false.

b) This is a true statement: For any angle t, it is the case that $\cos^2 t + \sin^2 t = 1$. Dividing by $\cos^2 t$ on both sides gives $1 + (\sin^2 t)/(\cos^2 t) = 1/\cos^2 t$. Since $1/\cos t = \sec t$ and $\sin t/\cos t = \tan t$, this gives $1 + \tan^2 t = \sec^2 t$ or $\sec^2 t - \tan^2 t = 1$.

Note that I started from $\cos^2 t + \sin^2 t = 1$, which I knew to be true, and derived $\sec^2 t - \tan^2 t = 1$. It is *not correct* to start from $\sec^2 t - \tan^2 t = 1$ and derive $\cos^2 t + \sin^2 t = 1$.

c) This is a conditional statement, since it could be true or false depending on the values of x and y. If $x = y = 0$ it's false. If $x = y = 1$, it's true.

d) This is false. It may look like it's conditional because it depends on the value of x, just like in c), but the "(where x is a real number)" is implicitly telling you that the statement is true for all real x.

It isn't, because if $0 < x < 1$, $\log_7(x) < 0$.

Solow 1.4 a) The hypothesis is that n is a positive integer and S is the sum of the first n positive integers. The conclusion is that $S = n(n+1)/2$.

b) The hypothesis is that r is real and satisfies $r^2 = 2$. The conclusion is that r is irrational.

c) The hypothesis is that p and q are positive real numbers with $\sqrt{pq} \neq (p+1)/2$ and the conclusion is that $p \neq q$.

d) The hypothesis is that x is a real number. The conclusion is that $x(x-1) \geq -1/4$.

Solow 1.8 If A is false and B is false, then "A implies B" is true. If A is true and B is false, then "A implies B" is false. Therefore, if B is false, you must show that A is false in order to show that "A implies B" is true.

Projective Plane Proof We are given the following axioms:

Axiom 1. Given any two distinct points, there is a unique line incident to both of them.

Axiom 2. Given any two distinct lines, there is a unique point incident to both of them.

Axiom 3. There are four distinct points such that no line is incident to more than two of them.

We must show that there are at least 7 points.

Call the four points givent to us by Axiom 3 A, B, C, and D. By Axiom 1, there exist lines AB, AC, AD, BC, BD, and CD. By axiom 2, there is a point incident to both AB and CD. Call that point P. Similarly, let the point incident to both AC and BD be Q and let the point incident to both AD and BC be R.

It is the case that P is distinct from A, B, C, and D: Say P were equal to A. Then, since P is on line CD, A would be on line CD, contradicting Axiom 3. Similarly, Q and R are distinct from A, B, C, and D.

It is also the case that P is distinct from Q: If P were equal to Q, then P would be both on lines AB and AC. But this contradicts Axiom 2, since AB and AC would have more than one point incident to both of them: A and P. Similarly, Q is distinct from R and P is distinct from R.

Thus, A, B, C, D, P, Q, and R are all distinct, and there are at least 7 points.

Pythagorean Theorem Everybody was able to find a valid proof on the internet (`http://www.wikipedia.org` and `http://www.cut-the-knot.org` were popular sources.)