Math 304
Homework 8 Solutions

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D.9 A sequence $X = (x_1, x_2, \ldots)$ is not monotone increasing if and only if for some i, $x_i \geq x_{i+1}$.

D.11 In order to show that a sequence $X = (x_1, x_2, \ldots)$ does not converge to a value L, we must show that there is an $\epsilon > 0$ such that for all j there is a $k > j$ such that $|x_k - L| \geq \epsilon$.

Let $\epsilon = 1/2$. Note that $|x_i - 1| = (i-1)/i$ which is greater than $1/2$ if $i \geq 3$. Therefore, given any j, we may take k to be the maximum of $\{3, j+1\}$ and we are done.

D.12 a) Suppose that $Y = (y_1, y_2, \ldots)$ converges to y. Then for all ϵ there is a $j(\epsilon)$ such that for all $k \geq j(\epsilon)$, $|y_k - y| < \epsilon$.

In order to show that $-Y = (-y_1, -y_2, \ldots)$ converges to $-y$ we must find, for each ϵ, a $j'(\epsilon)$ such that for all $k \geq j'(\epsilon)$, $|-y_k - (-y)| < \epsilon$. However, since $|-y_k - (-y)| = |y_k - y|$, we may take, for each ϵ, $j'(\epsilon) = j(\epsilon)$.

b) Since X coverges to x and $-Y$ converges to $-y$, $X - Y = X + (-Y)$ converges by Proposition 41 to $x + (-y) = x - y$.