MATH 304 **Homework** **Due March 6, 2008**

Recall from the discussion in class that if there are two injective (one-to-one) maps $f : X \to Y$ and $g : Y \to X$ between sets X and Y, there is a subset $A \subset X$ such that $F : X \to Y$ is a bijection, where F is well-defined by

$$F(x) = \begin{cases} f(x) & \text{for} \quad x \in A \\ g^{-1}(x) & \text{for} \quad x \in X - A. \end{cases}$$

In the following, remember to prove your statements.

Problem 1. Let $X = Y = \mathbb{N}_+$ be the positive natural numbers $\mathbb{N}_+ = \{1, 2, 3, \dots\}$, and let $f : \mathbb{N}_+ \to \mathbb{N}_+$ and $g : \mathbb{N}_+ \to \mathbb{N}_+$ be defined by $f(n) = g(n) = n + 1$. Find the subset $A \subset \mathbb{N}_+$ such that F, as defined above, is a well-defined bijection. In other words for $n \in A$, $F(n) = n + 1$, for $n \in \mathbb{N}_+ - A$, $F(n) = n - 1$, and F is a bijection.

Problem 2. Do Problem 1, but with $f(n) = g(n) = 2n$.

Problem 3. Show that the cardinality of the point in the plane is the same as the cardinality of the points in $\mathbb{R}$ the line.

Problem 4. What is the cardinality of the set of bijections of $\mathbb{N}_+$ to itself?

Problem 5. Problem 11.1 in Solow.

Problem 6. Problem 11.17 in Solow.

Problem 7. Problem 11.26 in Solow.