2.1.5 #3 The two statements are not the same. One says "$\forall x \exists y \ldots$" and the other says "$\exists y \forall x \ldots$." They are therefore different syntactically, and you can see that it makes a difference since the first is true and the second is false.

2.3.8 a) There exists an $x > 50$ such that $f(x) \neq g(x)$.

b) For all w there exists an $x > w$ such that $f(x) \neq g(x)$.

c) There exists a number x such that for all $\delta > 0$ there is a number t such that $|x - t| < \delta$ and $|f(x) - f(t)| \geq 1$.

d) For all $\delta > 0$ there exists a pair of numbers x and t such that $|x - t| < \delta$ and $|f(x) - f(t)| \geq 1$.

e) There exists $\epsilon > 0$ and a number x such that for all $\delta > 0$ there exists a t such that $|x - t| < \delta$ and $|f(x) - f(t)| \geq \epsilon$.

f) There exists $\epsilon > 0$ such that for all $\delta > 0$ there is a pair of numbers x and t such that $|x - t| < \delta$ and $|f(x) - f(t)| \geq \epsilon$.

3.6.6 #2 You can conclude nothing about the 1000th person; they could be either male or female.

3.7.3 #4 Assume that for all $\delta > 0$ there exists t and x belonging to $[0, 1]$ such that $|t - x| < \delta$ and $|f(t) - f(x)| \geq 1$.

3.8.4 #7 Suppose we're given $x \in (0, 1]$. Let

$$\delta = \frac{x^2}{2(1 + x)}.$$

Note that we are allowed to choose this value since $x^2 > 0$ and $2(1 + x) > 0$, so $\delta > 0$.

Suppose that $t \in (0, 1]$ is such that $|x - t| < \delta$. Then

$$\left|\frac{1}{x} - \frac{1}{t}\right| = \left|\frac{t - x}{tx}\right| < \frac{\delta}{|tx|}$$

Note that tx is positive, since both t and x are.

Since $|x - t| < \delta$, $t > x - \delta$, and $\delta/(tx) < \delta/((x - \delta)x)$. Since $\delta < x^2/(1 + x)$, this is less than

$$\frac{x^2/(1 + x)}{(x - x^2/(1 + x))x}$$

which simplifies to 1.

4.3.15 #9 For this problem, there are some things to watch out for: First, f does not have to be injective for $f^{-1}[A]$ to be defined for subsets A of its range. It only has to be injective for $f^{-1}(x)$ to be defined for *elements* x in its range.

Second, $f[f^{-1}[A]]$ need not equal A. Take $f(x) = x^2$ and $A = \mathbb{R}$, for example.

Third, $f^{-1}[f[A]]$ need not equal A. Take $f(x) = x^2$ and $A = \{x \in \mathbb{R} \mid x \geq 0\}$ for example.

Fourth, $f[A \cap B]$ need not equal $f[A] \cap f[B]$. Take $f(x) = x^2$, $A = \{-1\}$ and $B = \{1\}$ for example.

a) By definition, $f^{-1}[A \cup B]$ is equal to $\{x \mid f(x) \in A \cup B\}$. Since an element is in the union of two sets just in case it is in one of the two, this is equal to

$\{x \mid f(x) \in A \text{ or } f(x) \in B\}$, which is equal to $\{x \mid f(x) \in A\} \cup \{x \mid f(x) \in B\}$. By definition, this is $f^{-1}[A] \cup f^{-1}[B]$