The following are some definitions and Theorems that you can use for the homework and to understand the discussion of cardinal numbers.

Definition: Let $f : X \to Y$ and $g : Y \to X$ be two functions such that for all $x \in X$, $g(f(x)) = x$. Then we say that f is an *injection (one-to-one)* and g is an *surjection (onto)*. If, in addition, f is a surjection, we say it is a *bijection*.

Definition: Two sets X and Y are said to have the same cardinality if there is a bijection $f : X \to Y$.

Definition: Let the cardinality of a set X be a and the cardinality of the set Y be b. We say $a \leq b$ if there is an injection $f : X \to Y$. We say $a < b$ if, in addition, there is no bijection between X and Y.

Definition: A set X is *countable* if there is a bijection $f : \mathbb{N} \to X$, where $\mathbb{N} = \{1, 2, \ldots\}$ is the set of positive integers. The set is X is also said to have cardinality $\aleph_0$.

Definition: A set X is said to have *cardinality c* if there is a bijection $f : 2^{\mathbb{N}} \to X$, where $2^{\mathbb{N}}$ is the set of subsets of $\mathbb{N}$.

Theorem 0.1 *A countable union of countable sets is countable.*

Theorem 0.2 *An infinite subset of a countable set is countable.*

Theorem 0.3 *The set of all integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is countable.*

Theorem 0.4 *The set of rational numbers $\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z} - \{0\}\}$ is countable.*

Theorem 0.5 *The real numbers $\mathbb{R}$ have cardinality c.*

Theorem 0.6 *The irrational numbers $\mathbb{R} - \mathbb{Q}$ have cardinality c.*

Theorem 0.7 *(Schröder-Bernstein) If a and b are two cardinal numbers, $a \leq b$ and $b \leq a$, then $a = b$.*