MATH 304 **Practice Problems** **February 27, 2008**

Problem 1. Problems 1 to 10 on page 20 in Lewin.

Problem 2. Prove that the number of polynomials with rational coefficients is countable.

Problem 3. A real number is called *algebraic* if it is the root of a polynomial with rational coefficients. Prove that $\sqrt{2}$ is an algebraic number.

Problem 4. Prove that there are a countable number of algebraic numbers.

Problem 5. A line given by the equation $ax + by = c$, where not both a and b are 0, is called a rational line if a, b, c are all rational. Show that a line is rational if and only if it has, at least, two distinct rational points on it.

Problem 6. Prove that there are only a countable number of rational lines in the plane.

Problem 7. State the precise definition of what it means for a real valued function to be continuous at a point x_0.

Problem 8. State the precise definition of what it means for a real valued function not to be continuous at a point x_0.

Problem 9. Prove that the floor function $f(x) = \lfloor x \rfloor$ is not continuous at $x = 0$.

Problem 10. Prove that $f(x) = x\lfloor x \rfloor$ is continuous at $x = 0$.

Problem 11. Show that the function $f : \mathbb{R} - \{1\} \to \mathbb{R}$ defined by $f(x) = (x-2)/(x-1)$ is injective. Is f surjective? If not, find a point not in the image of f.

Problem 12. Prove that the sum of the cubes of three consecutive integers is divisible by 3. Do this with induction and without induction.

Problem 13. Consider the statements. If they are true, prove it. If false give a counterexample.

(a) The sum of two prime numbers is never a prime number.

(b) The reciprocal of a nonzero number of the kind $z = a + b\sqrt{5}$, with a and b rational numbers, is a number of the same kind.

Problem 14. Construct the truth table for $\neg(A \Rightarrow B)$ and compare with $\neg A \Rightarrow \neg B$.