Senior Seminar Topics

2006-2007		
Discrete Probability ***Abra Brisbin*** The first few days introduced set theory and combinatorics. After that, they turned to probability distributions, expected value, and independence. In the second half of the course, they worked on conditional probability, Bayes' Theorem, the Monte Carlo method, and Markov chains. Throughout the course, they tackled questions involving applications of probability in biology, medicine, social policy, and everyday life.	**Game Theory** ***Jason Anema*** This course began with a study of matrix games and a proof of the existence of Nash equilibria. Students then studied decision graphs, which included backwards induction, uncertainty and multiple-person decisions, and as an example played "Indian Poker" in class. The next topic was the problem of maximizing utility in auctions with incomplete information. The course concluded with a consideration of voting schemes and coalitions.	**Revisiting Combinatorics** ***Jay Schweig*** Combinatorics was taken in new directions for the students, covering graph and tree enumeration, partitions, compositions, and generating functions. Responding to a request from the students, the second part of the session covered the beginnings of formal logic, including a treatment of sentential logic, a discussion of the completeness and incompleteness theorems, and a full proof of the compactness theorem for sentential logic.
2007-2008		
Combinatorics: Unusual Counting Problems ***Gwyneth Whieldon*** Students started with proofs of interesting Fibonacci identities, and then moved on to more general binomial identities, with the emphasis on using bijective counting arguments rather than induction or other proof methods. Lucas and Gibonacci identities were studied, and more difficult identities that combine Lucas and Fibonacci numbers were studied. Binet's formula using combinatorial and probabilitistic arguments was proved. Identities on simple or general continued fractions was studied. Students were introduced to Khinchin's constant and some of its more unusual properties.	**Group Theory** ***Jonathan Needleman*** This course emphasized symmetries of mathematical objects, such as geometric shapes and sets. Basic properties of groups were explored including subgroups, normal subgroups, and quotient groups. Lagrange's Theorem and the first isomorphism theorem were proved, and the Sylow Theorems were stated. For an end of the term group project the students decided to explore symmetries in M.C. Escher's artwork.	**Introduction to Knot Theory** ***Victor Kostyuk*** This session started with basic concepts of knot and link projections, ambient isotopies, and Reidemeister moves, and continued with simple link and knot invariants (e.g., linking number, tricolorability, and crossing number). Alexander and Jones polynomials were introduced, the latter defined in terms of the Kauffman bracket. Students examined the Dawker notation and algebraic tangles, closed braid representations of links, torus and satellite knots. The last few weeks were spent discussing surfaces and the Euler characteristic, leading to a definition of a knot's genus and construction of Seifert surfaces.

2008-2009		
Counting Problems & Generating Functions ***Saul Blanco*** Students looked at the connection between rational generating functions and linear recurrences, and used these to find closed formulas for the Fibonacci and Catalan numbers that were defined recursively as the solution to counting problems. Students explored bijections between objects that are counted by Catalan and Motzkin numbers. Other topics included composition, set and number partitions, their associated generating functions, and Euler's pentagonal number theorem. Unlike the generating functions connected to linear recurrences, students discovered that the generating functions associated with partitions are an infinite product and not an infinite sum.	**Cardinality** ***Matt Noonan*** This session extended the unit on generating functions and examined how generating functions can lead to a generalized notion of cardinality. Students applied generating functions to the construction of "nonstandard dice." Understanding these dice is closely tied to understanding cyclotomic polynomials, and this became the new theme of the course. The seminar moved on to study constructability of regular polygons by ruler and compass. Finally, after studying some basic number theory in the form of Fermat's Little Theorem, Wilson's Theorem and Euler's Theorem, the the RSA encryption algorithm was introduced.	**Isometries & Symmetries** ***Victor Kostyuk*** Students looked at isometries of the line and plane, and symmetries of figures in the plane. This led to groups of isometries or symmetries. They discussed the axioms a set needs to satisfy in order to be a group, and studied basic examples of groups, their properties, and geometric expression as symmetries or isometries. Injective and surjective functions were covered, as well as homomorphisms and isomorphisms. They explored kernels, normal subgroups, quotient groups, and the first isomorphism theorem. Seminar closed with a discussion of generators, relations, and free groups. Universal properties and commutative diagrams were introduced in this context.
2009-2010		
Group Theory: Rubik's Cube ***Jennifer Biermann*** Students first focused on determining the size of the Rubik's cube group. For this they learned about permutation groups, decompositions of permutations into disjoint cycles, and even and odd permutations. They investigated subgroups of the Rubik's cube group and Lagrange's Theorem. The last part of the unit touched on several subjects such as Cayley graphs and quotient groups. The students were not taught how to solve the Rubik's cube but rather, time was spent discussing conjugates and how they could be used to find useful sequences of moves. Most students learned (on their own) how to solve the cube by the end of the unit.	**Paradoxes and Infinity** ***Gwyneth Whieldon*** This seminar began with discussion of several classical math paradoxes, properties of numbers and number systems, and the development of axiomatic mathematics. Students examined sizes of finite and infinite sets and several classic puzzles and paradoxes (e.g., Zeno's paradox, the Banach-Tarski paradox, the Littlewood Ping-Pong ball problem, and other Supertask problems). Other topics included partial fractions and Khinchin's constant, Fibonacci numbers and counting problems, and properties of infinite sums. Students took an historical look at the idea of "numbers," an historical examination of paradoxes of set theory, and the axiomatic systems of Whitehead and Russell.	**Introduction to Cryptology** ***Benjamin Lundell*** Basic Caesar and Rail Fence ciphers opened the seminar. After introducing modular arithmetic, students generalized to multiplication and, eventually, affine ciphers. Students studied monoalphabetic substitution ciphers using the Keyword cipher, and the cryptoanalysis of ciphers through frequency analysis and letter distributions. They learned ways to make more secure ciphers, which led to the Vigenère cipher and the one-time pad. They studied cryptoanalyis of polyalphabetic ciphers via the Friedman test, public key cryptography, and the Diffe-Hellman Key Exchange Protocol. We ended with a week long "cipher scavenger hunt," in which students cracked a series of ciphers leading to a prize hidden in the room.

2010-2011		
Counterexamples in Mathematics ***Mircea Pitici*** This seminar was based exclusively on analyzing, constructing, and discussing counterexamples in mathematics. We proceeded gradually, starting with counterexamples pertaining to basic functional notions and quickly advancing to counterexamples related to functional properties studied in calculus (continuity, differentiability, Darboux property, integrability). Among many examples we included some of historical importance (e.g., Dirichlet function and its variants, Weierstrass function). Most examples concerned functions of one variable, but toward the end we also studied counterexamples in functions of two variables. The initial intent was to include all branches of mathematics, but we decided to stay just within calculus for the whole seminar. However, as a final project, one student studied counterexamples in number theory.	**Probability** ***Tilo Nguyen*** The first segment of this seminar was an introduction to probability, starting with some refresher combinatorics problems. Students then learned how to use combinatorics and Venn diagram to calculate probability. We talked about the meaning of independent or mutually exclusive events. We discussed conditional probability and Bayes' Theorem, using disease testing as an example. We studied popular discrete distributions (e.g., Bernoulli, binomial, negative binomial, geometric, and Poisson distributions). We studied Markov chains and briefly discussed the use of Markov chains in real life applications (e.g., Google searches). The seminar ended with solving fun and famous probability problems (e.g., Buffon's needle).	**Numerical Analysis** ***Amy Cochran*** The seminar began with preliminary topics: computer arithmetic, error, logic, and basic programming using graphing calculators. The bulk of the seminar was focused on linear and nonlinear systems of equations. For linear systems, the students examined solution techniques (e.g., Gaussian Elimination, Backward/Forward Substitution, and Jacobi method), and learned about matrix and vector norms, singular value decomposition, and LU decomposition. For nonlinear systems, root-finding and minimization techniques were studied, including Newton-Raphson, bisection, golden search, and steepest descent methods. Numerical calculus was also briefly studied. Topics included Newton-Cotes formulas, Gaussian, and Monte Carlo integration.
2011-2012		
Special Curves ***Mircea Pitici*** We explored various special curves, with an emphasis on geometric elements; occasionally we also studied the algebraic and trigonometric properties, and pointed out the importance of the curves in applications, for instance in the theory and construction of mechanisms. All along we considered other curves related to a given curve, such as pedal curves and envelopes. We started off with a unified view of conics offered by projective geometry, mentioning several major closure theorems (due to Pascal, Brianchon, Poncelet) and a potpourri of side results. We continued by examining in detail the cycloid, a few particular epicycloids and hypocycloids (cardioid, astroid, deltoid, nephroid), and various spirals. Pressed by time, we mentioned cursorily some of the properties of limaçon, lemniscates, and ovals.	**Axiomatic Development of Probability** ***Mark Cerenzia*** This seminar presented the axiomatic approach to probability theory so that students could learn how mathematical machinery is built and applied. We began with a brisk introduction to relevant set theory in order to state the three axioms of probability (i.e., the definition of a measure space). We then derived typical properties one would expect when computing probabilities and showed how the framework helps us avoid pitfalls that both laymen and professionals often make. This led naturally into other core concepts, such as independence, conditional probability, Bayes' Formula, and random variables along with their important quantities (variance and expectation). Deriving everything formally from the axioms was the main feature and focus of this development.	**Calculus of Variations** ***Anoop Grewal*** We started off with the historical beginning of the subject with the famous Brachistochrone problem by Johann Bernoulli. The general solution by Euler and Lagrange was derived and discussed next. We discussed many famous applications of calculus of variations in engineering and physics, including geodesics on the plane, cylinder and sphere; Lagrangian formulation of mechanics; and the catenary curve as minimum potential energy solution.