MATH MATTERS

DEPARTMENT OF MATHEMATICS ❖ CORNELL UNIVERSITY ❖ ITHACA NY WINTER 2000

LETTER FROM THE CHAIR, JOHN SMILLIE

Last May, I presided over my first commencement reception as chair of the department. It was a moving experience, reminding all in attendance of the way that the department touches people's lives. This was also our first commencement in Malott Hall, our new departmental home. All the graduating seniors were asked to say a few words upon receiving their diploma, and all but one took advantage of the opportunity. Their comments ranged from "It meant a lot to me," to "The Math Department's the best!" to "I'd like to thank my roommate for doing my homework."

Once again, our department members have received their share of honors and awards. Professor John Hubbard was named by one of the university's undergraduate Merrill Presidential Scholars as the "faculty member who had the most positive influence" on his education at Cornell. It should be noted that this is the second year that John has been recognized in this way. Last February, Professor Yulij Ilyashenko was elected as president of the Independent University of Moscow. Graduate student David Brown won the Clark Distinguished Teaching Award. Daniel Ramras, an undergraduate major, was one of four Cornell recipients of a Goldwater Scholarship. The Harry L. Kieval Prize in Mathematics, an undergraduate award, was given to Karl Papadantonakis, who graduated summa cum laude last May. Our newly established Freshman Math Prize went to Prabhdeep Singh last May.

Though most of us are not doing research for the purpose of obtaining press coverage, it is nice to note those cases in which the popular science press picks up on mathematical activity related to Cornell. Professor Bob Connelly's recent work was featured in an article in the September 23, 2000 issue of *Science News*, entitled "Unlocking Puzzling Polygons," which can be viewed online. (See page three for additional information.) Another faculty member whose work received international attention this past year is H. C. Wang Assistant Professor Warwick Tucker. Tucker's article, "The Lorenz Attractor Exists," appeared in the French journal *Comptes Rendus*. (Corresponding article on page two.)

In September, the department began its first year of a three-year NSF VIGRE grant. This award will result in a broad range of new endeavors. It brings us new VIGRE assistant professors and new support for graduate students. It allows some graduate students to experiment with interdisciplinary activities or new teaching initiatives. It also supports increased involvement of undergraduates in research projects. Professor Rick Durrett describes our many VIGRE-related endeavors in his article on page three. Other VIGRE supported activities described in this newsletter are the enhanced Summer 2000 Research for Undergraduates (REU) Program on page four, the Saturday morning Math Explorers Club for area high school students on page two, and the new Interdisciplinary Colloquia for faculty and graduate students.

We continually benefit from the contributions of our young H.C. Wang assistant professors, and this year we are fortunate to also have four VIGRE assistant professors. This year's new H.C. Wang assistant professors are Warwick Tucker (mentioned above), who received a Ph.D. from Uppsala University, Sweden, and Jose Ramirez, who received a Ph.D. from New York University in May 2000, where he was a graduate student at the Courant Institute of Mathematical Sciences. Our four VIGRE assistant professors are James Conant, who received his Ph.D. from the University of California, San Diego in May 2000; Russell Miller who received his Ph.D. in June 2000 from the University of Chicago; Lawren Smithline, who received his Ph.D. in May 2000 from the University of California, Berkeley; and Edward Swartz, who received his Ph.D. from the University of Maryland, College Park in 1999. In addition, Harrison Tsai, a June 2000 Ph.D. from the University of California, Berkeley and a recent recipient of an NSF Postdoctoral Fellowship, has joined the department beginning this year for the duration of his fellowship.

An Old Problem Solved

Nearly four decades ago, Edward Lorenz introduced a simplified model of atmospheric dynamics in his now famous article "Deterministic Nonperiodic Flow" published in the *Journal of Atmospheric Sciences*. The simple system of differential equations produced amazingly complicated solutions. One stunning property was that solutions starting very close together were separated at an exponential rate. This gave rise to the concept of the "butterfly effect" and seriously undermined the idea of a deterministic world.

A 2D-view of the Lorenz attractor

Since Lorenz published his article, hundreds of articles have appeared addressing the peculiar system of differential equations. Although the area of dynamical systems had been steadily undergoing fundamental developments since the 1960's, Lorenz's equations defied all attempts at proving that they do exhibit a so-called "strange attractor" (see Smale's list of outstanding open problems for the next (this!) century published in *The Mathematical Intelligencer*).

A positive solution was announced about two years ago by Warwick Tucker, then a graduate student at Uppsala University, working under the supervision of Lennart Carleson. By using a novel combination of pure and applied mathematics, Tucker managed to prove that the equations do indeed give rise to a strange attractor.

Moreover, the attractor is robust, i.e., all nearby systems will display similar strange attractors. The proof has two main ingredients: rigorous numerics, which produce information about the global behavior of the system, and normal form theory, which deals with subtle local properties of the solutions.

Tucker's article "The Lorenz Attractor Exists" (available at www.math.uu.se/~warwick/papers.html) appeared in the French journal *Comptes Rendus*, and has been described in other journals such as *The Mathematical Intelligencer* (by Marcelo Viana) and in *Nature* (by Ian Stewart).

Math Explorers Club

This fall, the department began a new program for high school students, the Math Explorers Club. The Saturday meetings are open to all interested students at no charge. The goal of the club is twofold: to stimulate an interest in mathematics by exposing students to material not usually encountered in the high school curriculum and to provide a peer group of students who share an interest in mathematics.

Participants get to choose among modules that provide instruction in interesting areas of mathematics, computer lab activities, and problem solving sessions. Two intense hour-long sessions are separated by a break, during which participants of all modules can get together to relax and talk things over. The program is run by four graduate students (L. Gibson, S. Goel, M. Sloughter and T. Tomita) and has modules taught by faculty members. Typically there are two 6-week modules offered at a time, allowing new students to enter the club several times during the year. This fall the modules are Secret Codes and Cryptography (Ravi Ramakrishna), Fractals (Robert Strichartz), Math and Art (Robert Connelly and Maria Terrell), and Symmetry and Complex Numbers (Yulij Ilyashenko). Four more modules will be offered in the spring.

Currently there are more than 20 students from area school districts, and some home schools, who regularly attend, with almost equal gender representation. The majority of the participants are in tenth grade, but all high school grades are represented. Typically, there is a lot of competition for a high school student's free time, so we are encouraged by the turnout. We hope to hold on to current members until they graduate, as well as attract new students through word-of-mouth.

The program is funded by the VIGRE grant, which provides stipends for the grad students and pays for the refreshments. This is a great opportunity for the grad students to interact with faculty in an informal setting, and to gain valuable experience in education and outreach that will help them in future job searches. Participants enjoy the challenge of communicating the excitement of mathematics to young and eager minds, and spreading the word that mathematics is a cool subject. We expect the program to evolve and grow in response to the needs and interests of the participants.

Math Department Wins VIGRE Award

As the 1999 edition of *Math Matters* was going to press, the department was awaiting the outcome of its VIGRE grant application. In May 2000 we were awarded $1,450,000 over three years. This National Science Foundation program provides support to entire departments to rebuild the infrastructure of American mathematics. The acronym VIGRE stands for Vertically InteGrated Research and Education. Its goals are to prepare undergraduate students, graduate students and postdoctoral fellows for the broad range of opportunities available to individuals with training in the mathematical sciences and to promote the interaction of scholars across boundaries of academic age and department standing. There are now 26 of these grants nationwide, averaging five hundred thousand dollars per year for five years.

The first activity to benefit from the new VIGRE grant was the Research Experience for Undergraduates Program. The extra money allowed the program to be expanded to include three more Cornell undergraduates and provided additional financial support for the faculty members who devote eight weeks of their summer to supervising the projects. (Related article on page 4.) In addition, three undergraduates were supported by VIGRE to work directly with Cornell faculty members, doing research that will in some cases result in a senior honors thesis. Another impact VIGRE has on undergraduates is that all students can now benefit from the Smorgasbord series

(Continued on page 6)

Straightening Pernicious Planar Polygons

Polygons in the plane can be as regular as a square or as convoluted and irregular as a maze. People in the field of computational geometry have puzzled for some years over how to move the polygon into a standard shape, such as a triangle or a rectangle, or open the polygon into a convex shape, with no indentations or dimples, without changing the length of the edges or crossing edges over each other. This problem impacts the field of robotics, among others. If you can continuously move a closed polygon that closes back on itself to be convex, then it can be proved that a planar polygonal arm can be opened until it is straight.

This problem became a game within the field. Someone would propose a particular polygonal shape that was thought to be locked so that its shape could not change much from its starting position without changing edge lengths or crossing edges. Though the motions to open the polygon seemed to be tricky, it always turned out that it could be opened. Indeed, there were examples of five polygonal arms joined at a single vertex, which could be proved were locked. But this was not a single arm or a closed polygon.

In December 1999, at a conference in Budapest, Robert Connelly (Cornell) and Erik Demaine (Free University of Berlin) solved the problem. They proved that a closed polygon can always be moved until its shape is convex and that a polygonal arm can always be opened until it is straight without creating any self-intersections and without changing the lengths of the edges. These results were described in the September 23, 2000 edition of *Science News* in Ivars Peterson's article, "Unlocking Puzzling Polygons." (Available online at http://www.sciencenews.org/20000923/bob1.asp.)

Their findings were particularly helped by two important insights. Günter Rote's crucial observation in July of 1999 was that while it may be difficult to find, there always seemed to be an expansive motion that moved all pairs of vertices apart, one that opened the arm or made the polygon convex. If there is an expansive motion, it is easy to see that it never creates any self-intersection. The second critical observation was that the theory of rigid and flexible frameworks, a specialty of Connelly's, could be brought to bear. This provided the proof that there was always an expansive motion. The idea is to analyze the nature of stresses in the framework that is associated with the polygon. The picture below shows some of the stages in the opening of a sawtooth polygon.

"Teeth"
by Erik Demaine.
See his website for Animation.
http://daisy.uwaterloo.ca/~eddemain/linkage/animations/#teeth

Research Experiences for Undergraduates

The summer of 2000 was the seventh year of the Research Experiences for Undergraduates (REU) Program. The program is funded through a grant from the National Science Foundation. Last year, for the first time, the program was enhanced by funds from the VIGRE grant. All together, 14 undergraduates participated, including five from Cornell, assisted by three graduate students and several other visitors. The research areas and their faculty mentors were: 1) Analysis on Fractals with Robert Strichartz, 2) Mathematical Problems from Biology with Rick Durrett, and 3) Algebraic Combinations, with Richard Ehrenborg and Margaret Readdy. Ehrenborg and Readdy are former Cornell faculty who are now at the University of Kentucky.

Analysis on Fractals

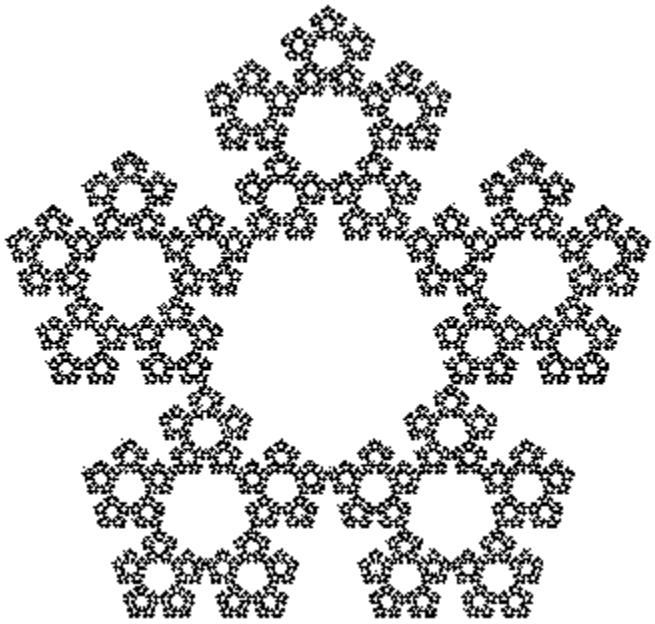
Since 1996, REU students have been doing research on fractals with Prof. Strichartz and Alexander Teplyaev (a 1998 Cornell PhD who is currently an NSF postdoctoral fellow at University of California, Riverside). The goal of the project is to create a kind of calculus for functions defined on fractals, following up on ideas originated by Jun Kigami. Already the cumulative work of the REU students has had a major impact on the field (the expository article "Analysis of Fractals" *Notices* AMS **46** (1999), 1199-1208, describes this area). Some of the accomplishments of the summer 2000 students are the following:

Bryant Adams (Washington and Lee) and **Alex Smith** (Cornell) worked on creating numerical analysis algorithms on the pentagasket (see figure below) and used these algorithms to begin exploring the spectrum of the Laplacian on this fractal. **Greg Padowski** (Cornell) created programs to study harmonic mappings of the Sierpinski gasket into the 2-sphere and the hyperbolic plane. He produced counter-examples to naïve conjectures that would have oversimplified the theory, and found numerically the boundary between uniqueness and non-uniqueness for stable energy minimizing maps into the 2-sphere. **Jeremy Stanley** (Wichita State) discovered a remarkable energy partition principle on fractals that has no analog on smooth manifolds. On fractals, as well as on Euclidean space, it is natural to write the total energy of a function as a sum of directional energies. On Euclidean space the directional energies are independent of each other, but in fractals each directional energy must be a fixed proportion of the total energy. **Andy Yingst** (University of North Texas) studied level sets of harmonic functions on the Sierpinski gasket and showed that generically all level sets are built up in a statistically identical way from small pieces.

Mathematical Problems from Biology

The students worked with Prof. Durrett and graduate students Janet Best and Peter Calabrese, in consultation with Cornell biologists in 3 areas:

Predator-prey models. **Robert Reynolds** (University of New Hampshire) and **Frank Yeager** (Cornell) studied differential equation models, asking the question: "How many prey species can coexist in an environment with a predator species?" The simplest models indicated an upper bound of n+1, but by introducing more sophisticated (and biologically reasonable) interactions they were able to construct models with one



PENTAGASKET

predator and more than 2 prey species coexisting.

Chromosome size evolution. **Arkendra De** (Purdue), **Michael Ferguson** (Cornell) and **Suzanne Sindi** (Cal State Fullerton) studied a model of Sankoff and Ferretti to predict the distribution of sizes of chromosomes and the changes over time in the distributions. There were serious discrepancies between the predictions of the model and biological data. They were able to modify the model to obtain a better fit with the data from 8 species.

Microsatellites. **De**, **Ferguson**, and **Sindi**, working with recent Cornell graduate James Signorovich, studied the distribution of repeat DNA sequences (microsatellites) in the noncoding part of the genome for humans and C. elegans. This portion of the genome tolerates a much wider variation among individuals than the coding portion (genes), since mutations are usually harmless, and so it is useful in evolutionary studies and DNA fingerprinting. The mathematical models help to test the biological assumptions about the types of mutations that occur.

Algebraic Combinatorics

Graduate student Leah Gold assisted students working with Ehrenborg and Readdy on 4 individual projects. **Debbie Grier** (Cornell) studied the flag vector of hyperplane arrangements in Euclidean spaces of arbitrary dimension. The flag vector encodes all the combinatorial properties of containment among the intersections of the hyperplanes. The *cd*-index of the flag vector further compresses this information into a polynomial in noncommuting variables *c* and *d*. She obtained a recursion relation that characterizes the *cd*-index for a special class of arrangements. **Adnan Rubai** (SUNY Binghamton) studied a variant of the game Nim played on a simplicial complex, where the rules for removing chips involve the structure of the simplicial complex. (The chips are placed in piles on the vertices of the complex.) He was able to characterize winning strategies for some examples. **Geir Helleloid** (Wisconsin) studied the structure of certain infinite groups of permutations related to juggling patterns. In particular, he was able to compute the generating function for the number of group elements expressible in terms of *n* elements from a standard set of generators for the group. **Michael Levin** (Harvard) studied certain conjectured inequalities relating to the number of ups and downs in a permutation (the descent set statistics). He had the brilliant inspiration to relate the question to a probability question involving ups and downs of a random sequence of numbers. This question led to conjectured integral inequalities for certain polynomials and their derivatives. He was able to prove several of these, thus verifying a number of the original conjectures for permutations.

Summer 2001 Program

The REU program continues in 2001. The projects will be Analysis on Fractals, directed by Robert Strichartz, Geometry of Numbers directed by Konstantin Rybnikov (an H.C. Wang assistant professor) and Computational Discrete Geometry, directed by Károly Bezdek (a visitor form Eötvös University in Budapest.)

Summer Research Minigrants

The VIGRE program also provides small grants during the summer to undergraduates who wish to do research with faculty members not involved in the REU program, with the expectation that this work will lead to writing a senior thesis. In 2000, these grants were awarded to three students: **Jesse Alt**, working with Sergei Artemov in logic; **Anselm Levskaya**, working with John Hubbard on Maxwell's equations; **Chan-Ho Suh**, working with Ravi Ramakrishna on number theory. These grants will again be available for summer 2001.

The Math Table

The Math Table is designed to bring undergraduates interested in mathematics together with other like-minded students, as well as graduate students and faculty. This successful program was initiated by Prof. Graeme Bailey several years ago and is sponsored by the Office of Campus Life.

The Math Table is modeled after Cornell's Language House program, in which students get intensive exposure to a different language and culture. In this case, an average of 25 participants gather each Friday evening in Risley dining hall to discuss and share ideas related to mathematics, as well as to join an active social group with ongoing debates about politics and upcoming social events. It is an invaluable opportunity for undergraduates to interact with faculty outside of the classroom, and provides an open forum for sharing ideas and asking questions.

(VIGRE, CONTINUED FROM PAGE 3)

of talks from the REU program. The lecture series will now be available during the school year and carries 1 credit for attending the 13 lectures. A second activity that will have an impact on our undergraduates is the mandatory curriculum review associated with the grant. Changes are coming soon in the way we teach analysis, reorganizing our courses to make them more useful for our majors.

The fall semester saw the arrival of four new VIGRE postdocs, Russell Miller (Chicago), James Conant (UC San Diego), Ed Swartz (U of Maryland), and Lawren Smithline (UC Berkeley). The new postdocs have a teaching load of one course per semester to allow them time to continue to develop their research skills. They also have a faculty mentor to help them navigate the mysterious process of being a professional mathematician, such as applying for grants, learning how to teach, and dividing your time successfully between teaching and research.

Also new on the scene were our first four VIGRE graduate fellows, David Benbennick (U. of Alaska, Fairbanks), Kristin Camenga (a graduate of St. Olaf who has been a high school teacher for some time), Jeffrey Mermin (Duke), and Everilis Santana-Vega (U. of Puerto Rico, Humaco). These students will have their entire first year free from teaching responsibilities, and they will have one semester free in each of the second and third years to allow them to broaden their education and to explore possible thesis areas.

Another opportunity for current graduate students to benefit from the grant is through VIGRE semesters. These resemble fellowships in that students submit proposals to the committee and, if awarded a VIGRE semester, are relieved of teaching duty for a semester to pursue their own research. In the fall, Mohan Rajagopalan used his free time to take two courses in Computer Science, working towards a minor in that area. Kathryn Nyman worked with Dave Bock (visiting us from Ithaca High School) in Math 171 to prepare for teaching it herself in the spring. Suzanne Hruska assisted Beverly West in Math 103, a course on Patterns and Iteration for liberal arts students. Sharad Goel (Ctr for Applied Math) helped run the Math Explorer's Club, a Saturday activity for local high school students (related article on page 2). Maria Sloughter will, in the spring, work under the guidance of Allen Back to develop computer lab activities for the Math Explorer's Club. Lee Gibson will explore applications of mathematics to biology with Rick Durrett. Leah Gold will escape from the dreary Ithaca winter and spend a month at the University of Nice where faculty member Mike Stillman will be on sabbatical. Leah will learn about the applications of computer algebra to biology there, and at Cornell through discussions with Ron Elber and Jon Kleinberg in Computer Science. Both Lee and Leah will have VIGRE support to attend the short course on Mathematical Biology at the AMS meeting in New Orleans in January.

A second lecture series, the VIGRE Interdisciplinary Colloquium is aimed at graduate students, postdocs, and faculty. Eva Tardos from Computer Science gave the first talk on some interesting network flow problems. Steve Ellner from Ecology and Evolutionary Biology discussed a number of studies of population dynamics to argue that chaos rarely existed in nature. Steve Tanksley from Plant Breeding gave the final talk of the semester, "Genomics: The Fusion of Life Sciences, Computation and Engineering." Next semester will begin with a lecture by Steve Strogatz from Theoretical and Applied Mechanics and will feature a mini-symposium on "Statistics and the Law." Watch our web page www.math.cornell.edu/~durrett/VIGRE/ for more details on these lectures and the VIGRE program in general.

TA COORDINATOR POSITION FILLED

Maria Terrell has joined the department as teaching assistant coordinator and senior lecturer, effective July 1, 2000. She fills a position left vacant by Tom Rishel. For the past decade, Maria has been a part-time adjunct professor in our department, as well as a full-time assistant dean for admissions and advising in the College of Arts and Sciences at Cornell. We are very fortunate to have someone in this position with Maria's extensive experience in the student services area. Her many good ideas and enthusiasm for teaching have already served the department well.

Tom Rishel left Cornell at the end of last year to take the Associate Directorship of the Mathematical Association of America in Washington, D.C. where he's now in charge of programs and services. Tom coordinated our teaching assistant program for many years. He will be missed by the graduate student community, staff and faculty.

2000 Kieval Lecture

Colin Adams, the Francis C. Oakley Third Century Professor in the Department of Mathematics and Statistics at Williams College, gave the Harry S. Kieval Lecture at Cornell Sept. 29. The lecture, "Real Estate in Hyperbolic Space: Investment Opportunities in the Next Millennium," was held in Malott Hall.

Adams explained how our regular geometric notions of things like area are very different in hyperbolic geometry, but he did so with a twist. For his informative and humorous talk, Adams assumed the persona of tacky real estate agent Mel Slugbate. Mel 'pitched' real estate in hyperbolic space as opposed to the regular Euclidean space, along the way explaining the mathematics of this different geometry. (For instance, the area of a circle in hyperbolic space increases exponentially with the radius—there's more land to buy!) Adams assumed his role during the reception prior to the lecture, distributing his brochures to the approximately one hundred students and faculty in attendance.

Adams' lecture, designed for undergraduate students and members of the public who have a basic scientific and mathematical knowledge, was funded through a bequest of the late Dr. Harry S. Kieval '36, a longtime professor of mathematics at Humboldt State University in Arcata, CA, who died in 1994.

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❑**The Mathematics Faculty Book Fund.** Provides the Cornell community with immediate access to one of the world's finest assortments of math books and publications by enriching the collection of the Mathematics Library.

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