What is a graph?

- Vertex
- Edge
What is a graph?
What are graph colorings?

Edge Colorings

Vertex Colorings
The party problem

6 people
The party problem

3 mutual friends

Hello, friends!

3 mutual non-friends

Who are these people?
The party problem

Friends

Not friends
The party problem

3 mutual friends

3 mutual non-friends
The party problem
The party problem
The party problem
The party problem
The party problem
Ramsey theory

\[ R(m, n) = \min\{k \mid K_k \text{ with edges colored red/blue has either a red } K_n \text{ or a blue } K_m \} \]

\[ R(3, 3) = 6 \]
Ramsey theory

- Proven:
  - $R(2,n) = n$
  - $R(3,4) = 9$
  - $R(4,4) = 17$

- Proven through computation:
  - $R(3,8) = 28$
  - $R(3,9) = 36$
  - $R(4,5) = 24$
Ramsey theory

- General bounds:
  - $R(n, n) \leq 4^n$
  - $R(m, n) \leq R(m - 1, n) + R(m, n - 1)$
What are graph colorings?

Edge Colorings

Vertex Colorings
Vertex coloring
Application: map coloring

Chromatic number:
Minimum number of colors needed to properly color the graph
Vertex coloring

Problem: find the chromatic number of a graph
Greedy vertex coloring
Greedy vertex coloring

Color 1
Color 2
Color 3
Color 4

...
Greedy vertex coloring

Color 1
Color 2
Color 3
Color 4

...
Greedy vertex coloring
Greedy vertex coloring

Shows chromatic number is no more than max degree + 1
Greedy vertex coloring

A tight example:

A loose example:
Chromatic number

Are there any scalable graphs with low chromatic number?

Idea: limit graph structure to no triangles
Chromatic number: no triangles
Mycielski graphs

$M_2$
Mycielski graphs

$M_2$

$M_3$
Mycielski graphs

$M_4$
Mycielski graphs
Mycielski graphs

$M_4$
Vertex colorings

- Other sparse constructions of high chromatic number graphs
- Vertex colorings on hypergraphs
- Better algorithms to color vertices?
- Better bounds on chromatic number?
Resources

- Material: Budapest Semesters in Mathematics “Advanced Combinatorics Handout,”
  [https://pdfs.semanticscholar.org/03f5/e4c50ceda0303e46608939c3a6a48f488728.pdf](https://pdfs.semanticscholar.org/03f5/e4c50ceda0303e46608939c3a6a48f488728.pdf)
- South America map coloring: [https://mapchart.net/world.html](https://mapchart.net/world.html)
- Slide template: [https://www.slidescarnival.com/escalus-free-presentation-template/963](https://www.slidescarnival.com/escalus-free-presentation-template/963)
Thank you!

...Questions?

The shift graph $\text{SH}_5$