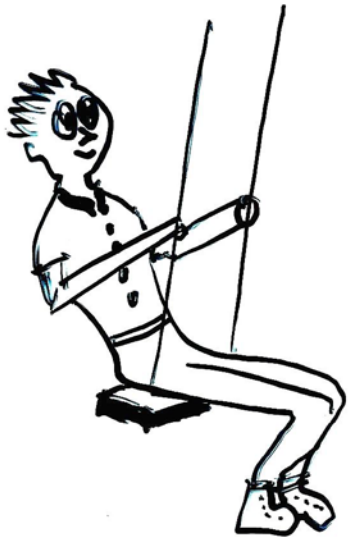
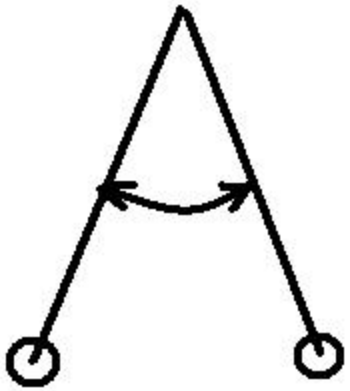
Modeling the Pumping of a Swing

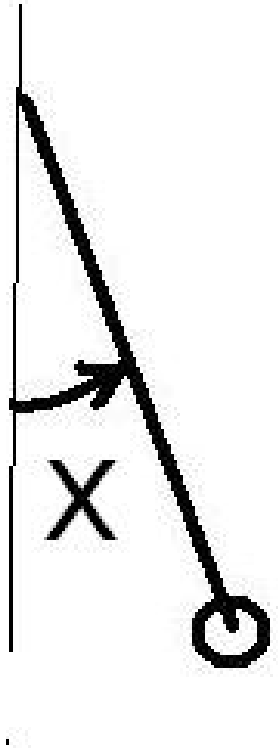


Richard Rand

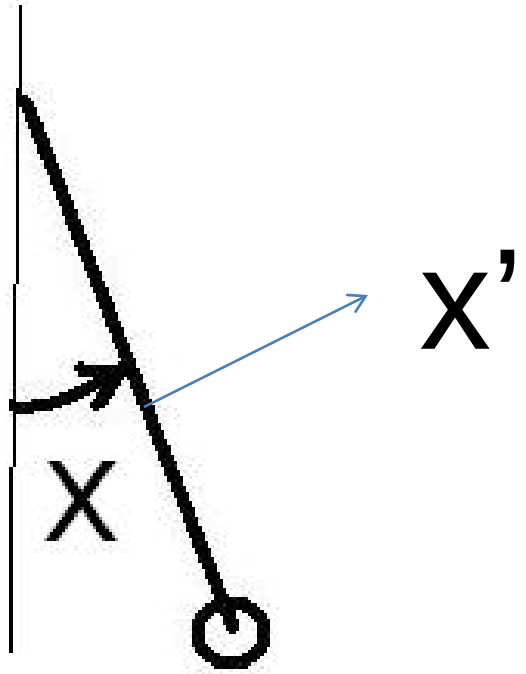
A swing is modelled as a Pendulum



The position of the pendulum is given by the angle x:

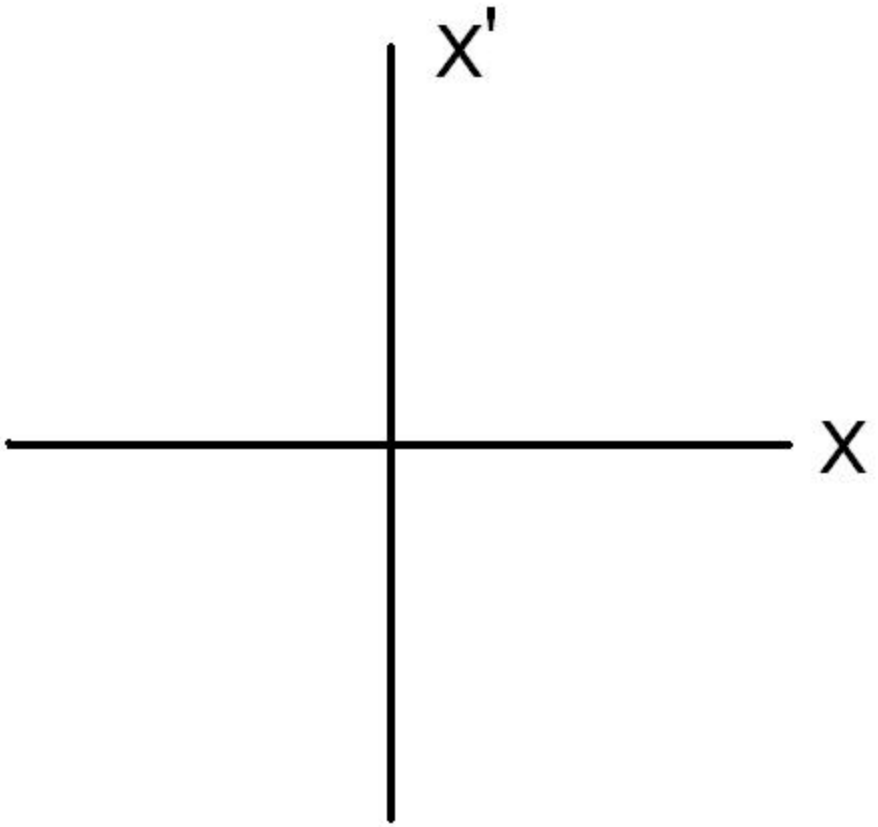


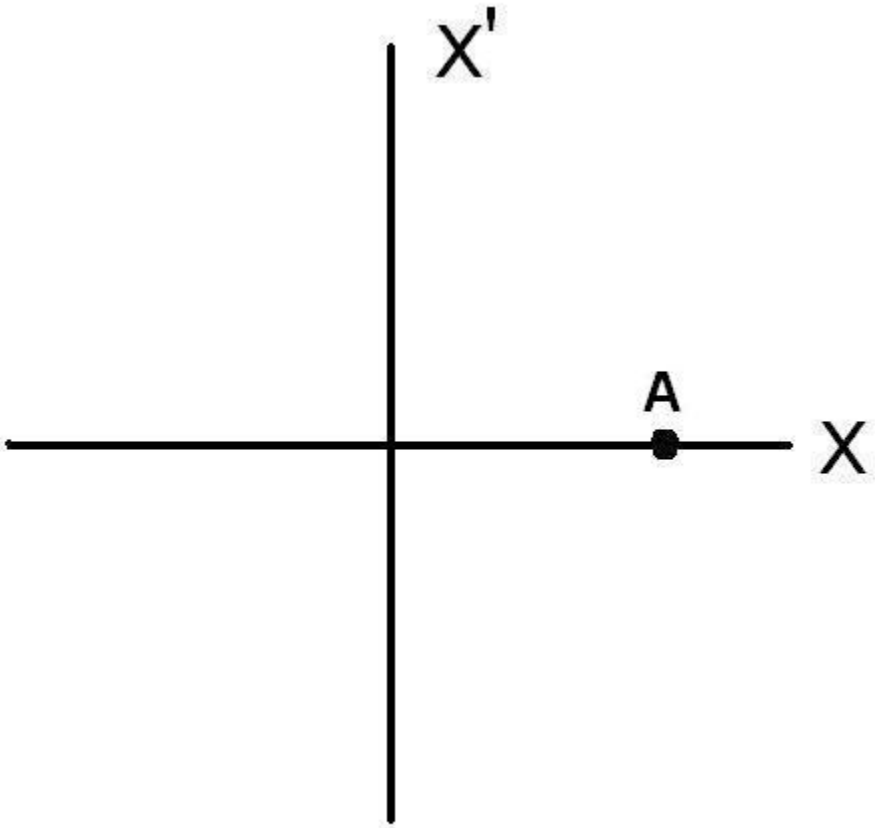
The position of the pendulum is given by the angle x:

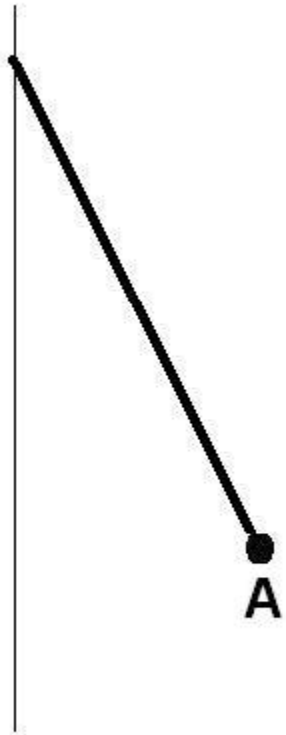


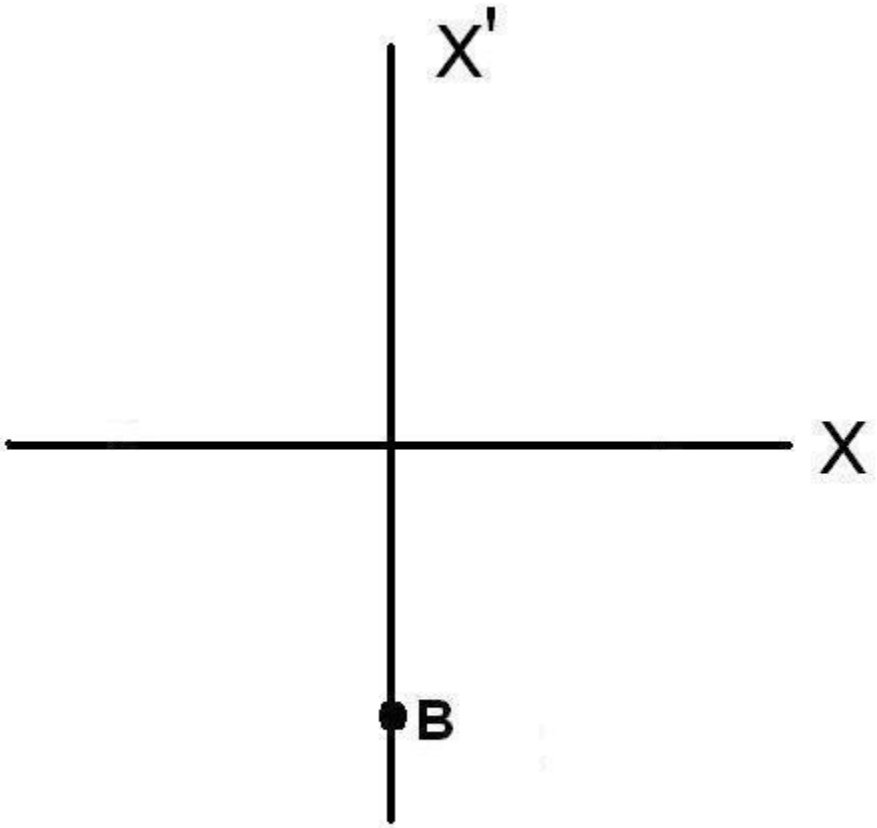
It's angular velocity is given by x'

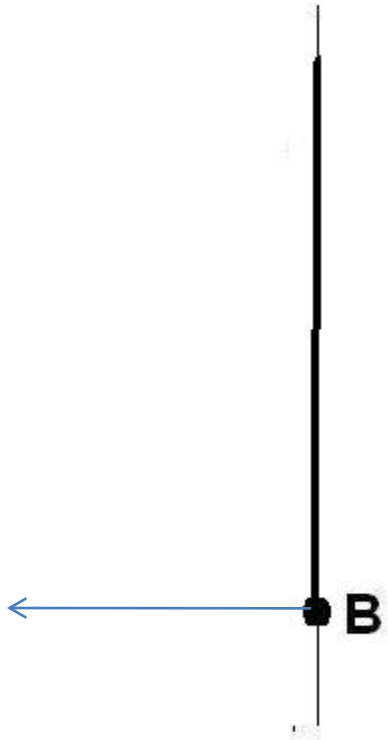
The “state” of the pendulum is given by the pair (x, x’)

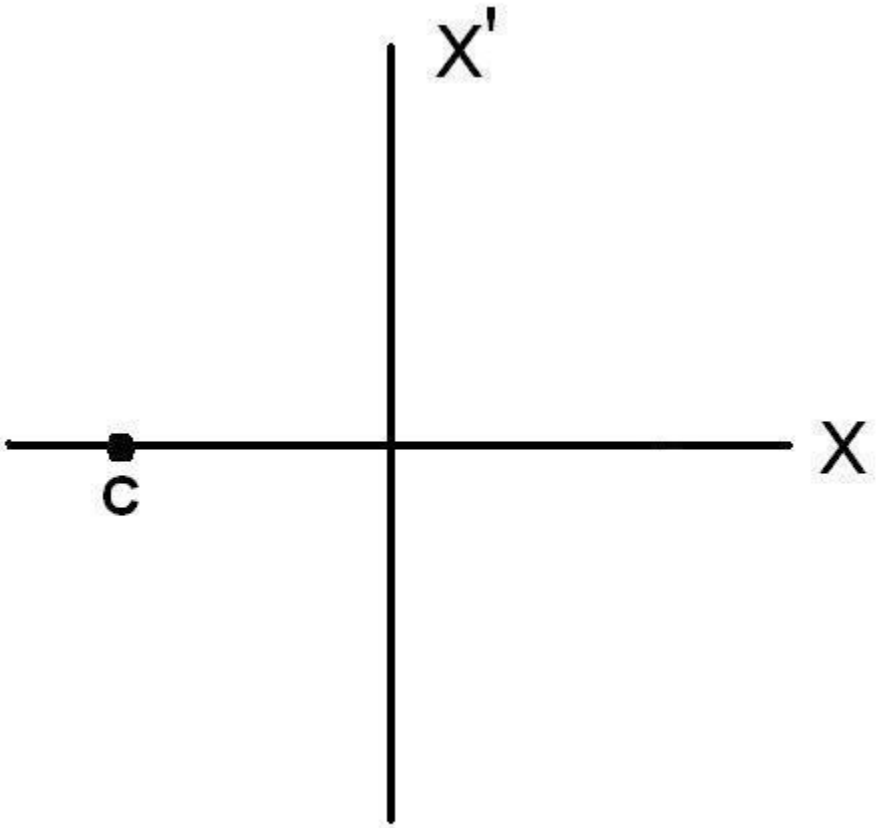


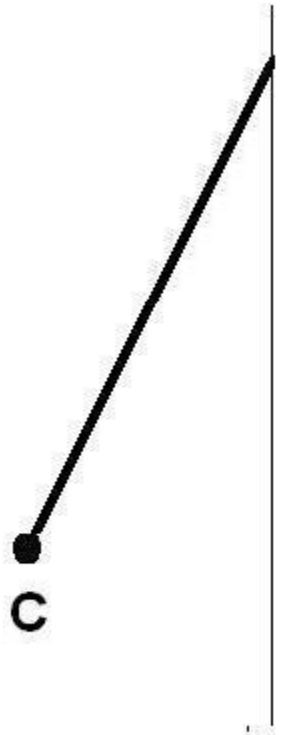

X'
A
X

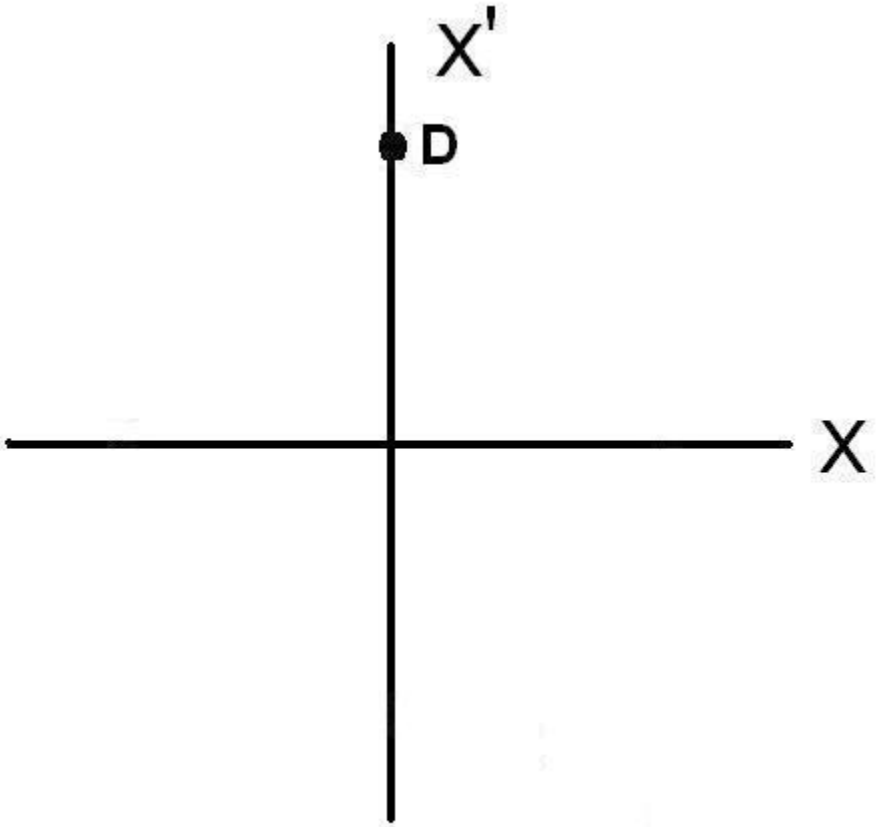

A

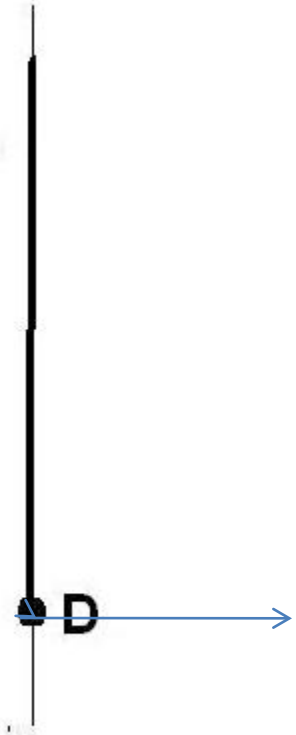

X'
X
B

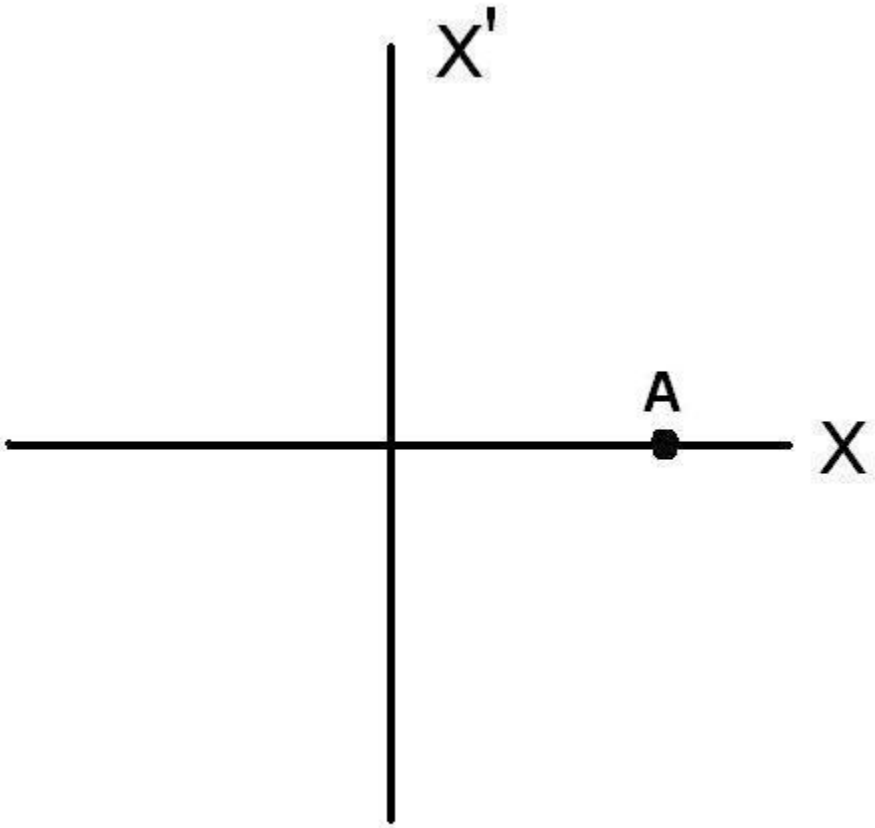

B

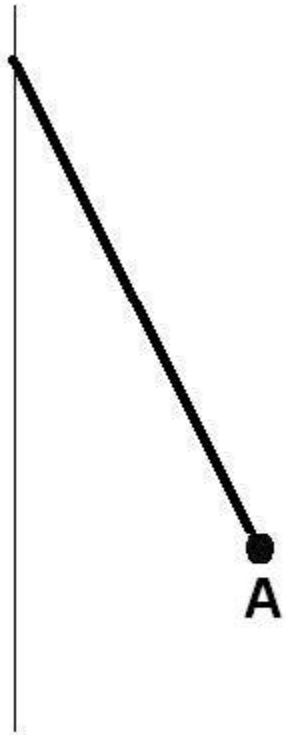

X'
X
C

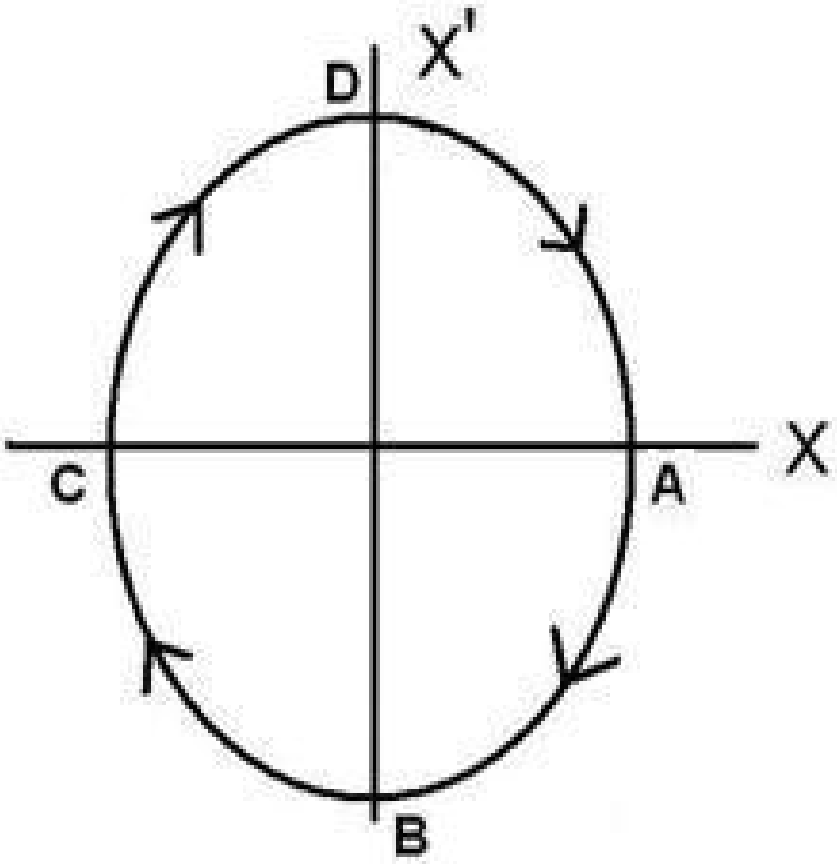

C

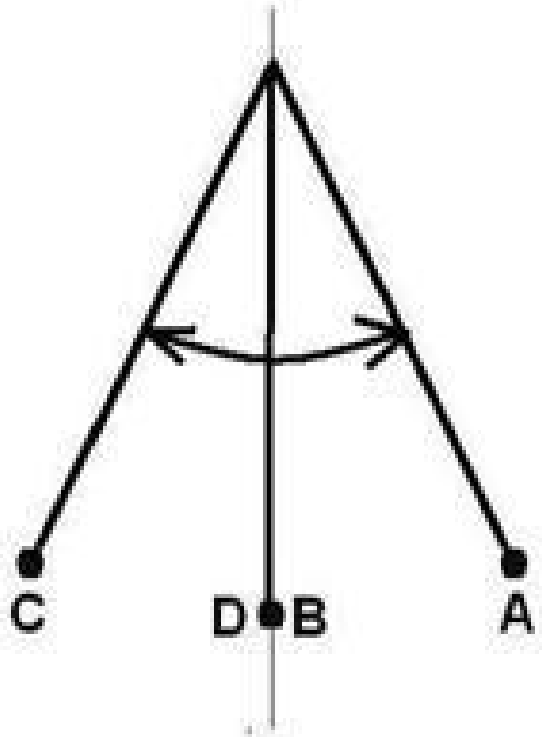

X'
D
X

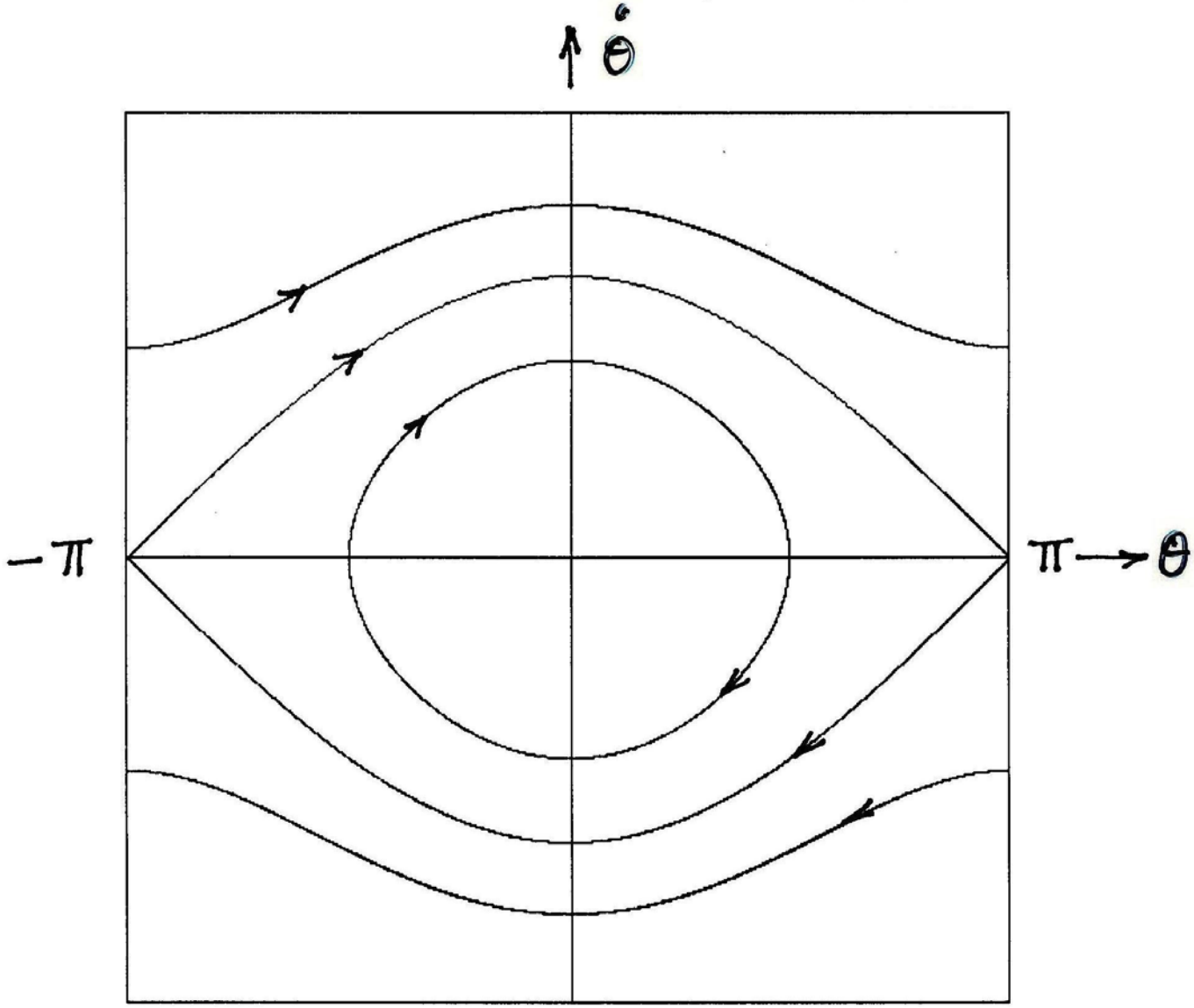

D

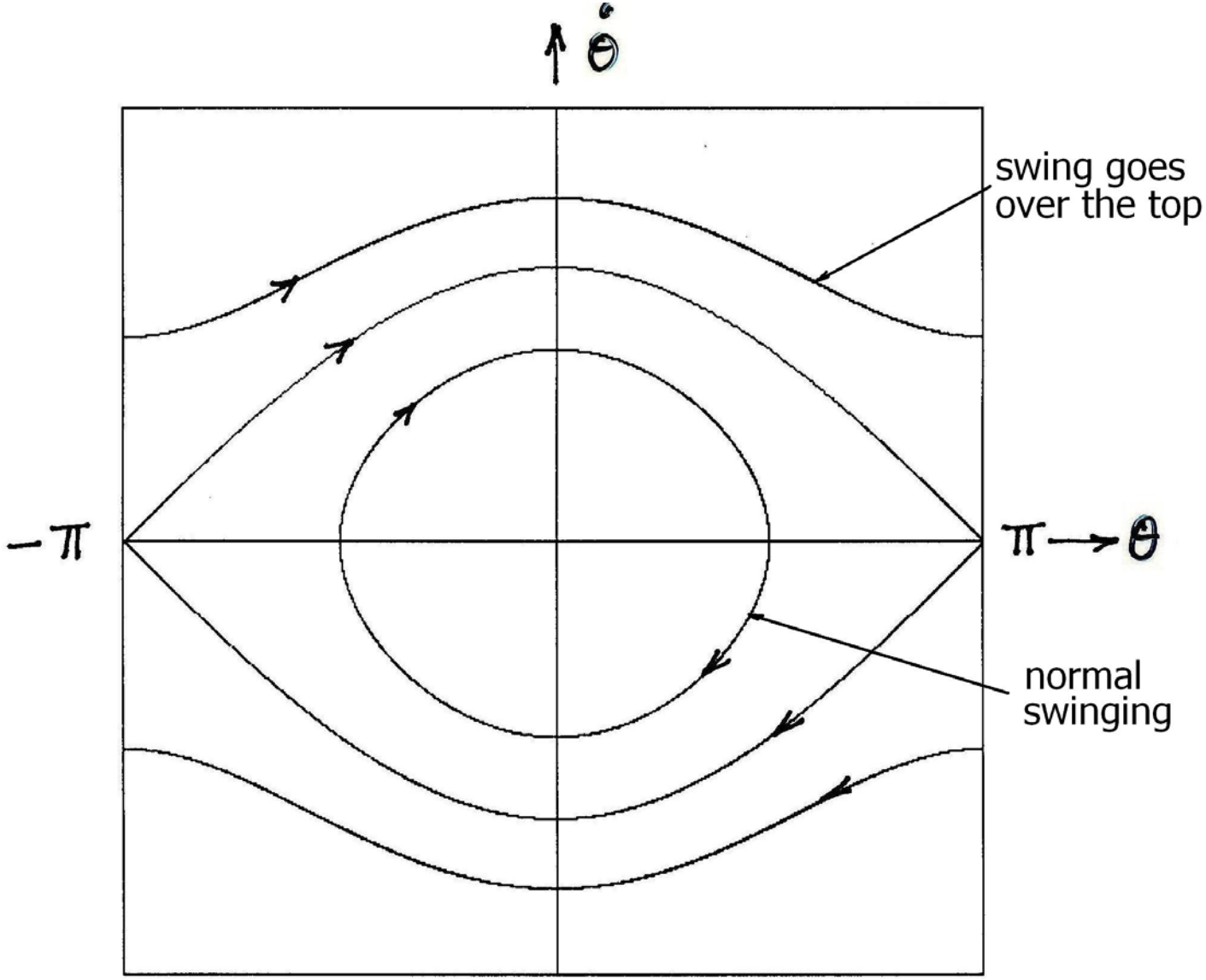

X'
A
X


A


X'
D
X
C
A
B

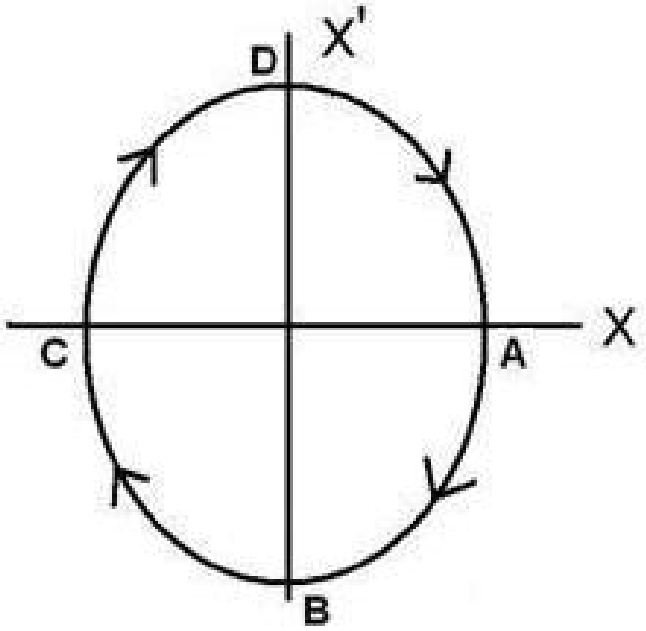

C
D
B
A

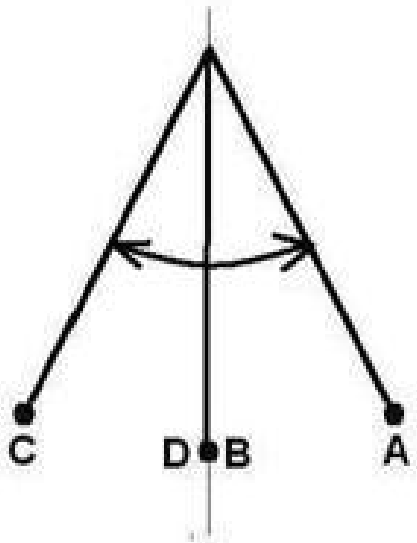

$\dot{\theta}$
$-\pi$
π
θ


$\dot{\theta}$
swing goes over the top
$-\pi$
$\pi \rightarrow \theta$
normal swinging

First method for pumping:

Standing rider
squats at A and C, and stands at B and D





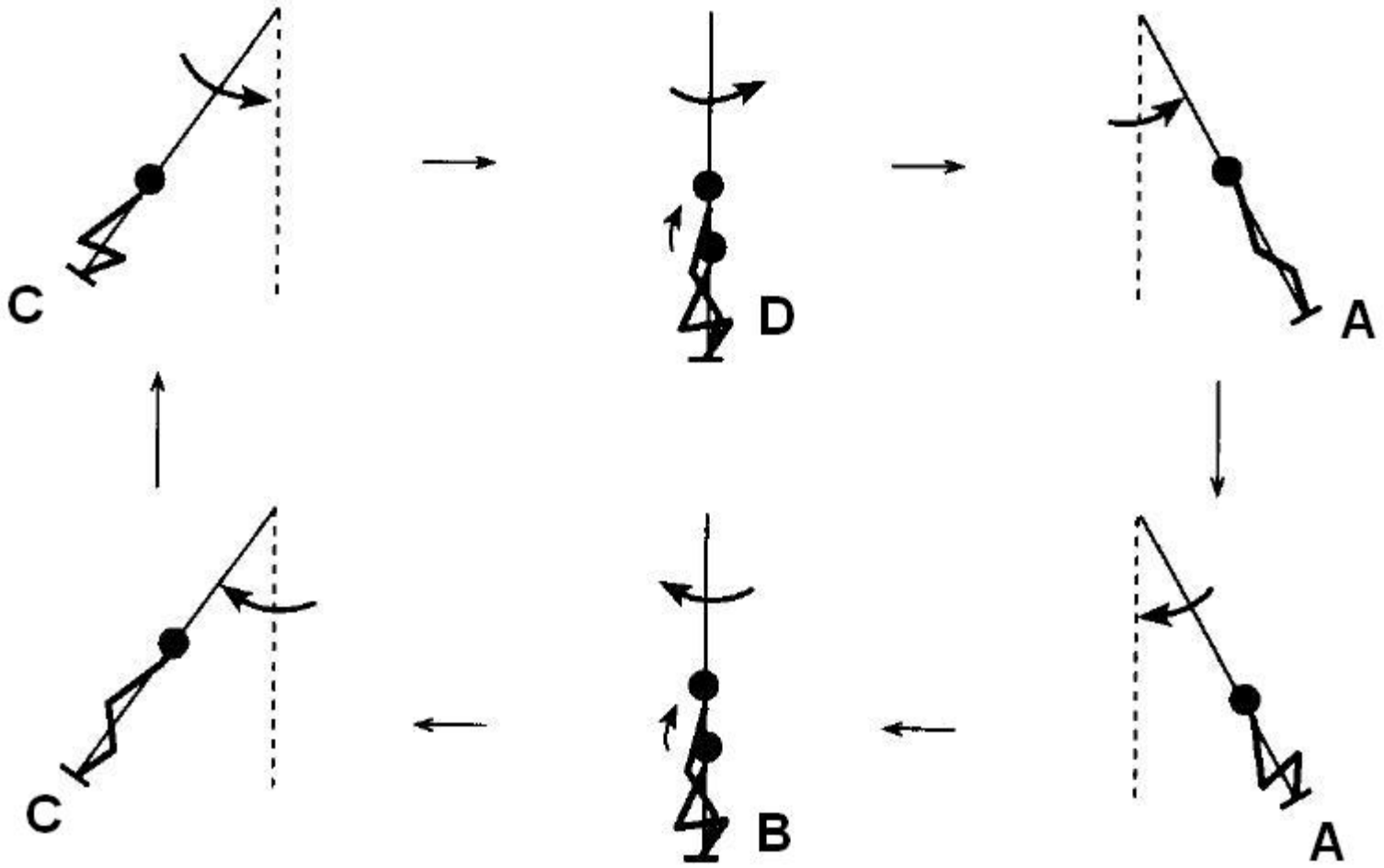


Figure 2. Pumping strategy for a standing rider.

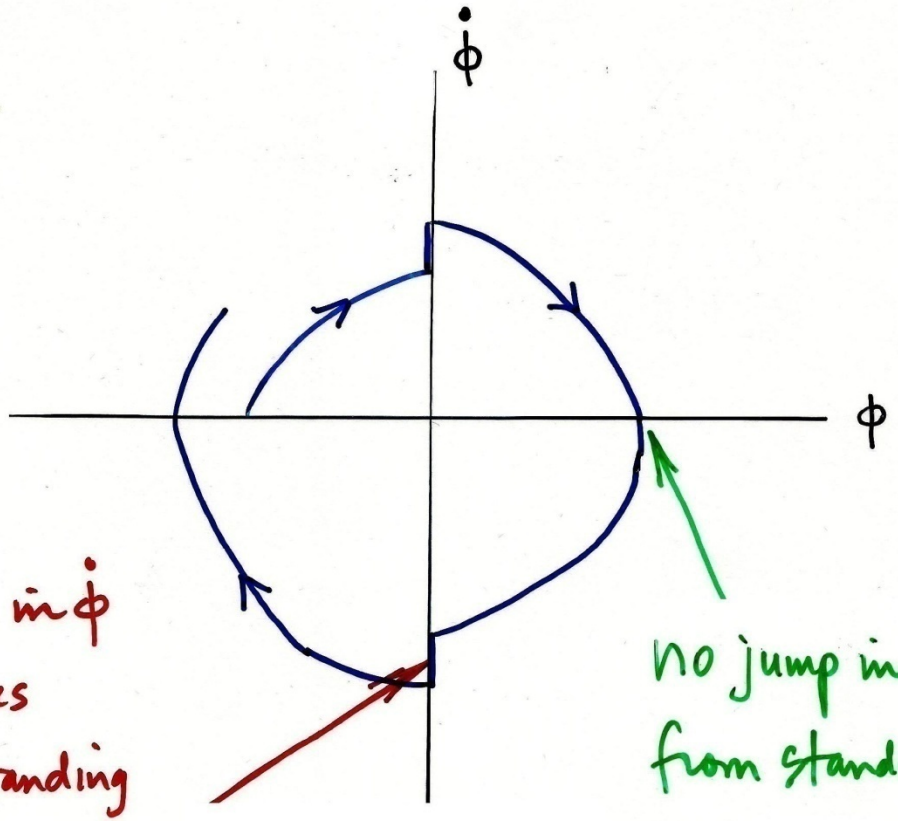
Physics of why it works:
Conservation of angular momentum

Angular momentum = L^2 x’

(L^2 before squat) (x’ before squat) =
(L^2 after squat) (x’ after squat)

x’ after = (L^2 before/L^2 after) x’ before

STANDING 2:1 PUMPING



jump occurs in $\dot{\phi}$ as rider changes from squat to standing

no jump in $\dot{\phi}$ as rider changes from standing to squat

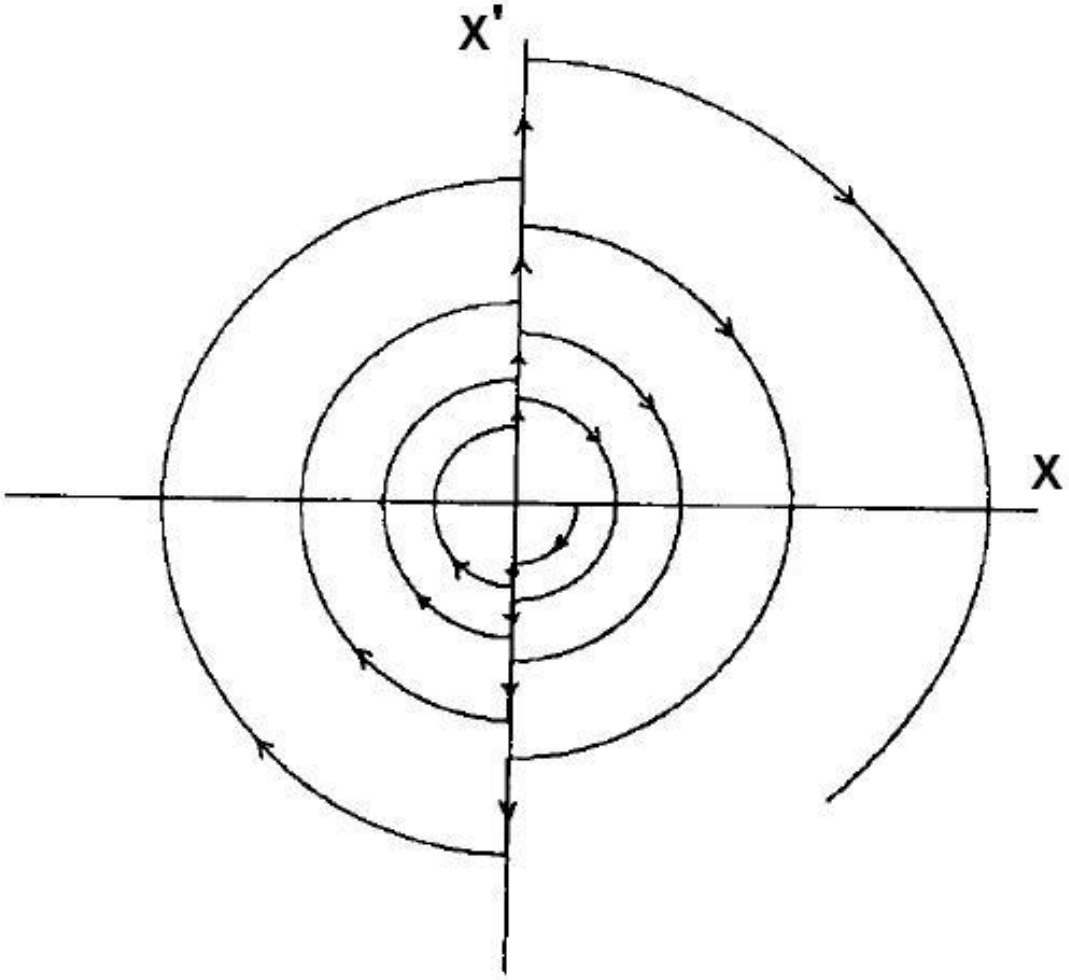
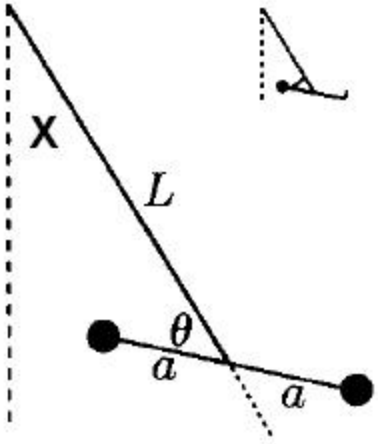


Figure 3. Phase trajectory for standing pumping. $L_{\text{stand}} = 2.3$, $L_{\text{squat}} = 2.7$

Second method for pumping:

Seated rider
rotates body at A and C.



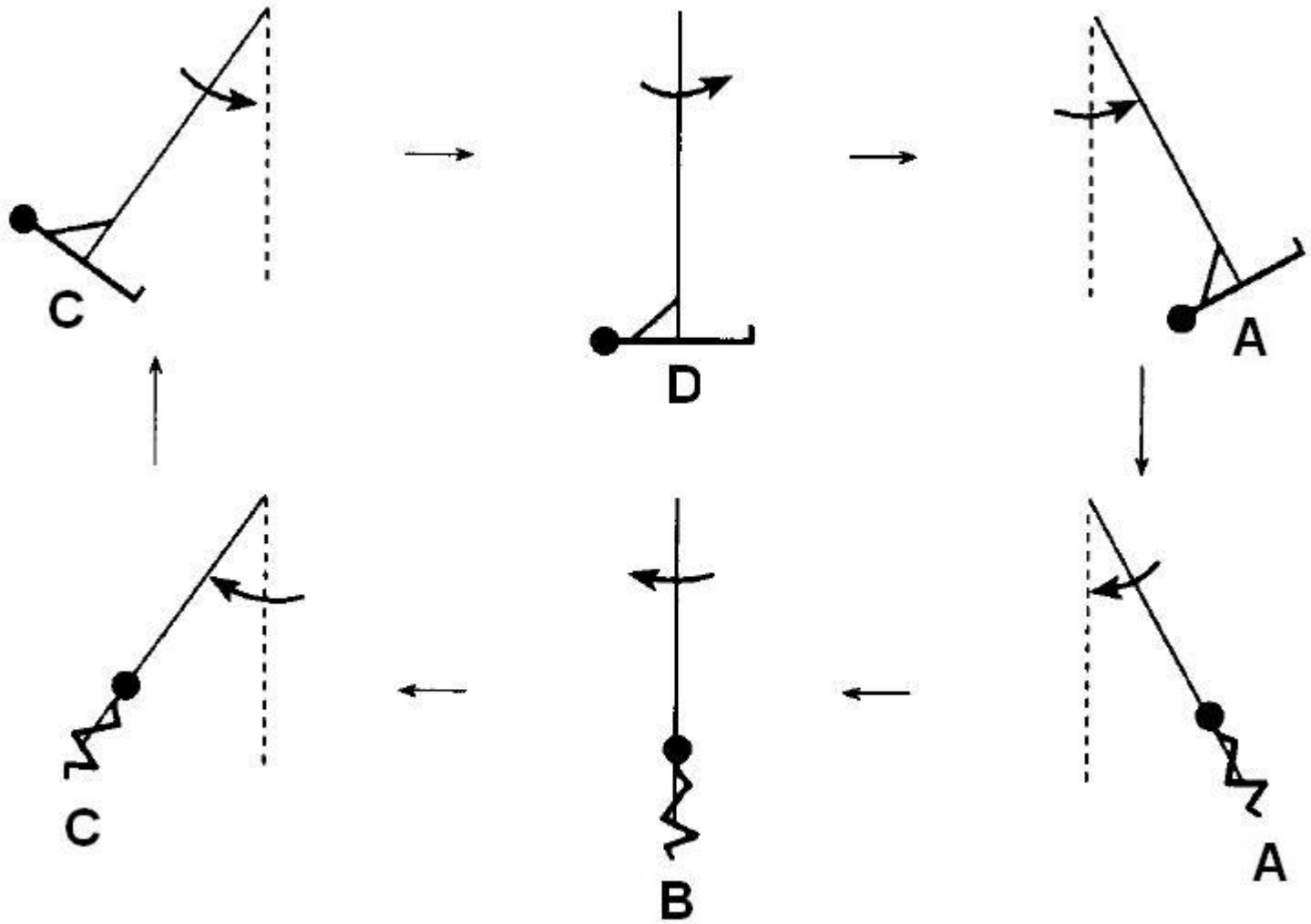


Figure 5. Strategy for pumping while seated.

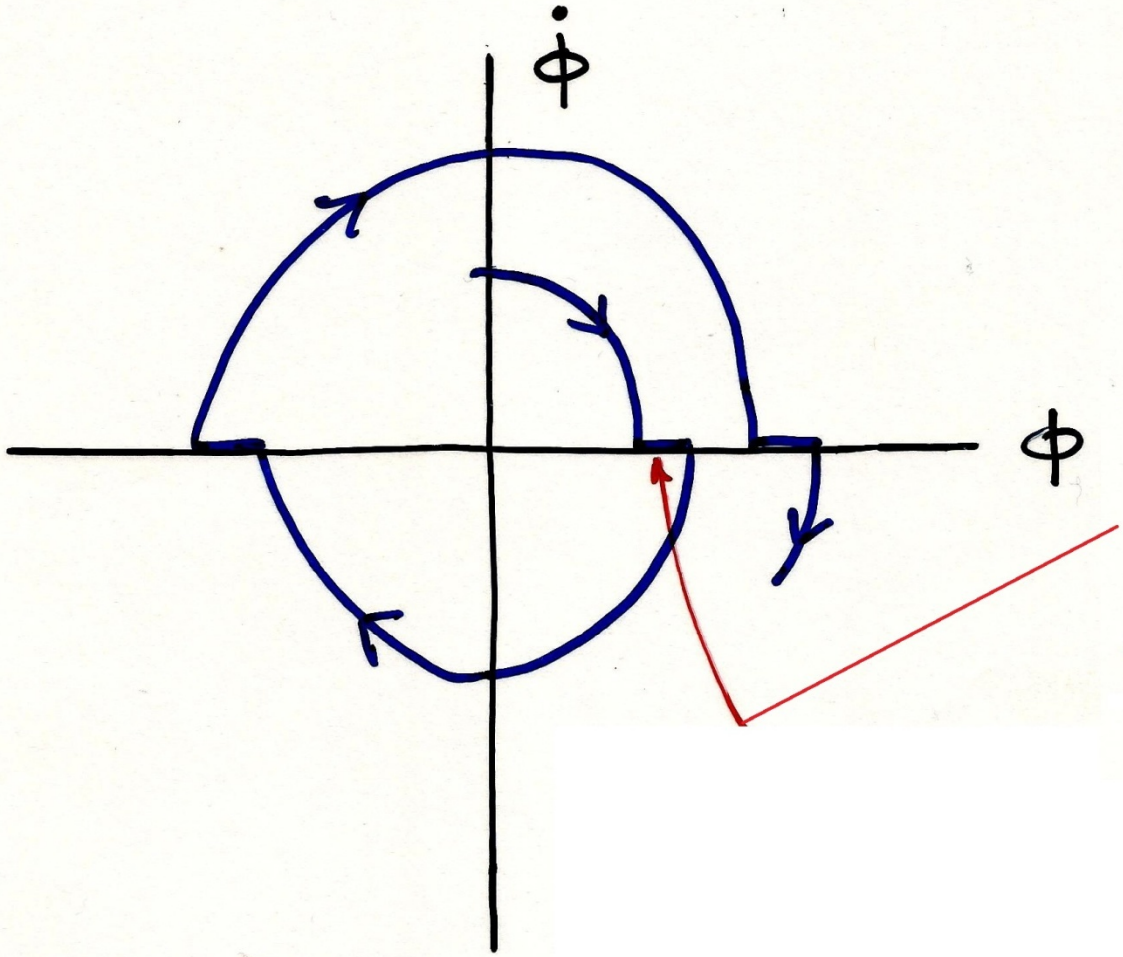
Physics of why it works:

Rotating body adds a burst of angular momentum to the system, call it M.

At A and C, the angular velocity is zero.

The result of M is to make x jump a constant amount.

SEATED 1:1 PUMPING



jump occurs in $\dot{\phi}$ as rider rotates her body

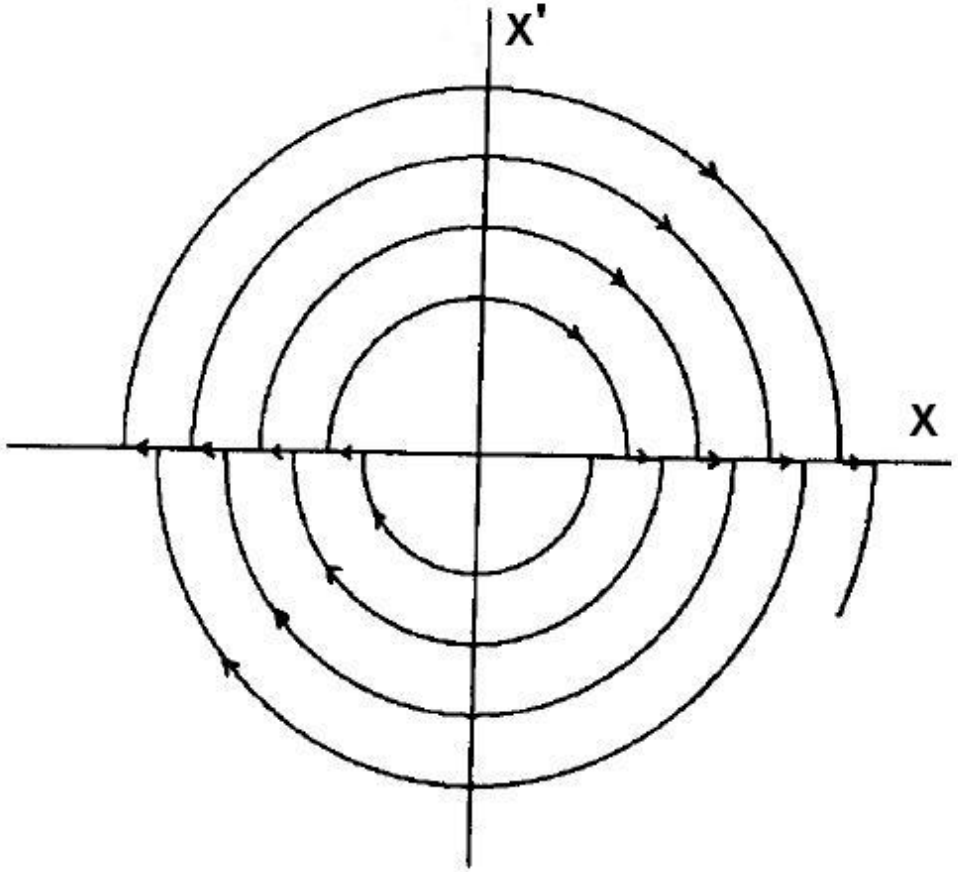
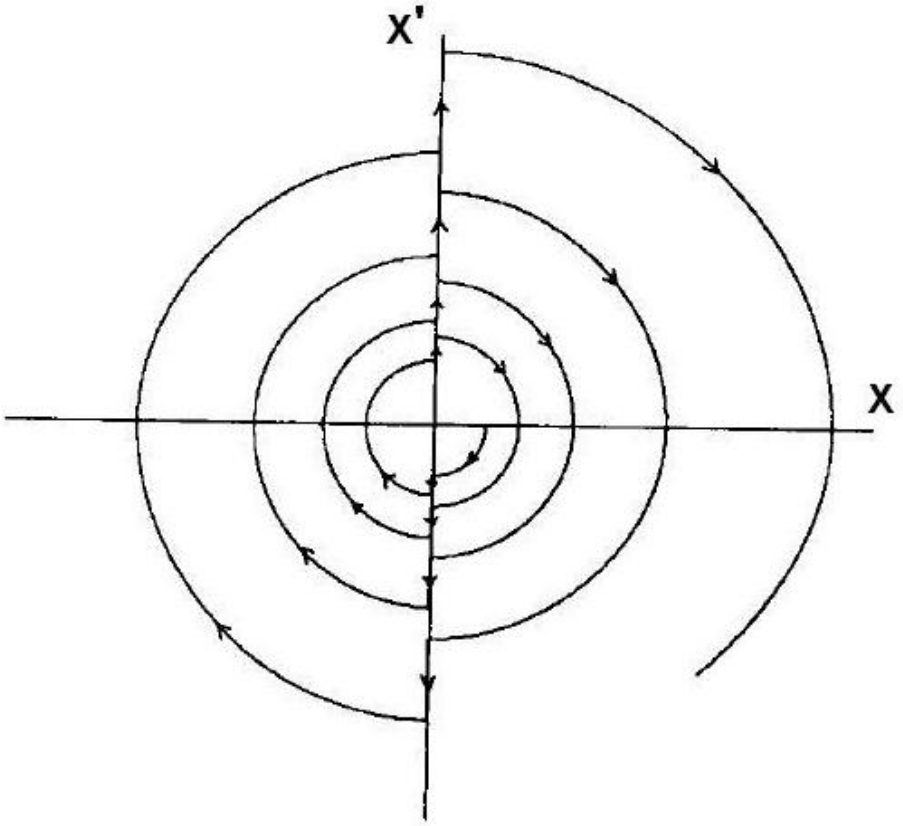


Figure 6. Phase trajectory for seated pumping. $L = 2.5$, $a = 0.5$.

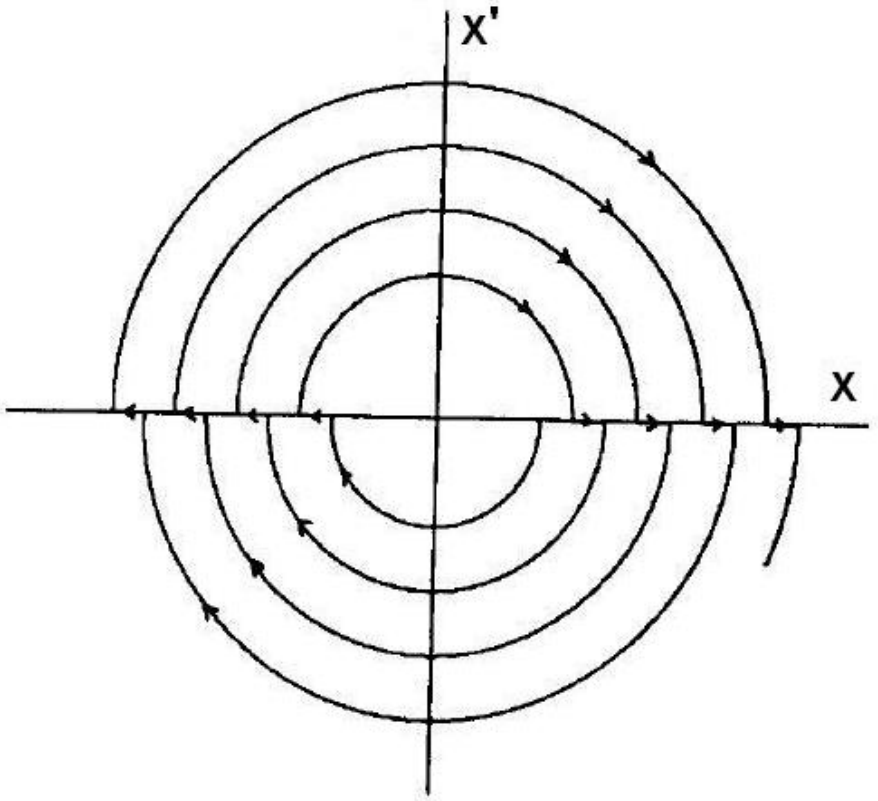
Comparison
of the two types of pumping

standing pumping



x'after = x'before * K,
where K>1

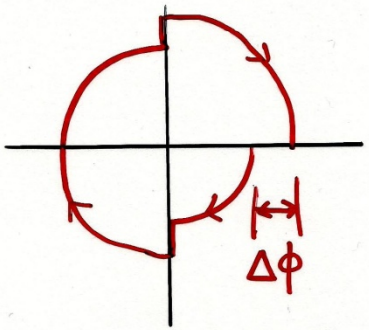
seated pumping



x after = x before + M
where M>0

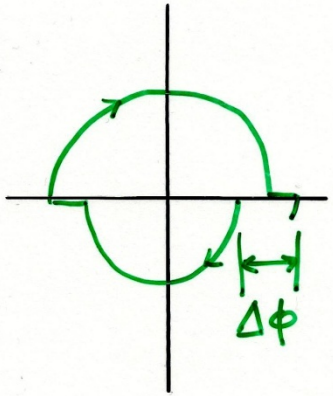
COMPARISON OF TWO TYPES OF FORCING

2:1 ΔL FORCING

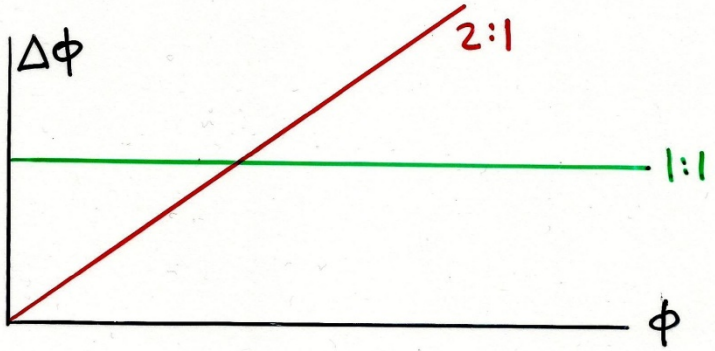


$\Delta\phi = \text{const} \cdot \phi$

1:1 $\Delta\theta$ FORCING



$\Delta\phi = \text{const}$



For largest $\Delta\phi$, 1:1 is best for small ϕ,
2:1 is best for larger ϕ.

In one case we multiply by a number K>1.

In the other case we add a number M>0.

Which is better?

Conclusion

Standing pumping multiplies the current amplitude by a factor larger than 1.

Seated pumping adds a fixed quantity to the current amplitude.

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Thus for small amplitudes, seated pumping is better.
But for larger amplitudes, standing pumping is better

Conclusion

Standing pumping multiplies the current amplitude by a factor larger than 1.

Seated pumping adds a fixed quantity to the current amplitude.

Thus for small amplitudes, seated pumping is better.
But for larger amplitudes, standing pumping is better

So start off by seated pumping, then switch to standing pumping when the amplitude gets large enough.

Reference

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S.Wirkus, R.Rand, A.Ruina

The College Mathematics Journal

29:266-275 (1998)