Summer 2019 Cornell REU:
Mechanics, Control, Robotics, and Dynamics

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This project will focus on the analysis of problems in mechanics, control theory, and robotics from the perspective of dynamical systems theory. Within mechanics, we will focus on the equilibrium and stability properties of thin elastic structures, and we will use these results to model the behavior of toys such as a Slinky. Within control theory, we will study problems in optimal control, inverse optimal control, and ensemble control. Within the field of robotics, we will focus on the problem of automated manipulation for deformable objects and on the problem of simultaneously controlling many robots with a limited number of signals. Finally, within dynamical systems theory, we will study the connections between stability and optimality and how optimality can be used to model collective motion.

Students participating in this project should have strong backgrounds in linear algebra and ordinary differential equations. Previous experience with computer programming, optimization, classical mechanics, or control theory will be helpful, but is not required. The problems we will solve can be tailored to students' expertise, and the problems can be computational, analytical, or a mix of both. Below are descriptions of potential topics that students can explore during this project.

Mechanics

- **Stability of thin elastic structures**

  Thin structures, such as a one-dimensional wire or a two-dimensional piece of paper, often respond in a nonlinear manner to external forces. This nonlinear response can result in phenomena such as buckling, creasing, crumpling, and other instabilities. Although engineers have spent centuries attempting to prevent instability, a recent trend has emerged in which these phenomena are exploited, a transition some have called Buckliphobia to Buckliphilia [1]. To fully utilize these nonlinear phenomena, a deeper understanding of the circumstances under which they occur is needed. This portion of the project will focus on finding conditions under which thin elastic structures can and cannot experience instabilities.

- **Mechanics of toys**

  The same toys that fascinate children can be equally fascinating to mathematicians. Explaining how a Slinky forms its iconic arch shape [2], how a toy "popper" jumps off a table [3], and how small magnetic spheres can be formed into long chains [4] requires mathematical tools such as optimization and differential equations. This portion of the project will focus on understanding the mechanics of toys. If you have a favorite toy that you would like to spend the summer modeling, bring it with you!

Control

- **Inverse optimal control**

  Optimization problems typically provide you with a cost function and ask you to find the choice that results in the lowest cost. On the other hand, an inverse optimization problem provides you with choices that are optimal and asks you to find the function that is being optimized. Once the cost function is recovered, it can be used to make optimal choices in future situations. Inverse optimal control is a type of inverse optimization problem in which the optimal choices are time-dependent trajectories. This portion of the project will focus on developing analytical and computational tools for solving inverse optimal control problems.

- **Ensemble control**

  Suppose you are given a dynamical system that contains many agents. These agents could be robots, cars, arial vehicles, etc. The motion of each agent is dictated by signals sent by you to the agent. Also suppose that there are some small differences between how the agents respond to your signal. For example, if you tell all the agents to move left 1 meter, some agents may move 0.9 meters, while others may move 1.1 meters. Ensemble control asks the following question: Is it possible to make the agents behave in a desired way while sending the same signal to all agents? This portion of the project will focus on developing methods to solve ensemble control problems and on applications in which ensemble control arrises.

Robotics

- **Robotic manipulation of deformable objects**

  Look inside any factory that uses robots to handle and assemble parts, and you're likely to see the robots interacting with objects that are nearly rigid. What you're less likely to see are robots interacting with objects that experience large deformations. Robotic manipulation of deformable objects appears to be challenging because the robot must reason about the infinite number of shapes the object can take. This portion of the project will focus on characterizing the set of all possible shapes of a deformable object and on developing methods that allow robots to efficiently manipulate a deformable object into a desired shape.

- **Steering robots by moving their targets**

  This problem is closely related to the ensemble control problem described above. However, rather than sending signals to the agents, suppose the agents move in the direction of a nearby target. What type of behavior can we generate by moving the target? With only one target, this problem is not very interesting, since it can be shown that all agents typically approach the same location [5]. Now suppose there are two targets that we can move, and the agents move in the direction of the nearest target. What behavior can be generated? Is it possible to maintain a minimum separation between each of the agents? This portion of the project will explore these and similar questions.

Dynamics

- **Collective motion**

  Flocks of birds, schools of fish, and swarms of insects are capable large-scale coordinated motion, sometimes called collective motion, even though each individual is only able to see what their nearby neighbors are doing. Heuristic models of this behavior have been proposed, which rely on assumptions about the goals of each individual in the swarm [6]. Is it possible that each individual is acting in a way that optimizes a cost function? This portion of the project will study how collective motion can be modeled from an optimization viewpoint and how the choice of cost function influences the behavior of the swarm.

- **The relationship between stability and optimality**

  Stability is a property of dynamical systems in which trajectories that begin close to each other remain close. Optimality is a property of dynamical systems in which trajectories are minimizers of some cost function. This portion of the project will explore the relationships between these two properties. Questions we will ask include: What is the relationship between stability and optimality of fixed points in a dynamical system? Are there conditions under which stability can provide information about the optimality of a trajectory? Conversely, are there conditions under which optimality can provide information about the stability of a trajectory?

References

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