2019 Upstate Number Theory Conference Schedule

All hour long talks will be in **251 Malott Hall**

SATURDAY:

8:30 - 9:00am	Registration
9:00 - 9:55am	Abbey Bourdon, Wake Forest University *Isolated Points on Modular Curves*
10:00am - 10:55am	Aaron Pollack, Duke University *Modular forms on* G_2
11:00 - 11:35 am	Coffee/Discussion Time
11:40 - 12:45pm	Contributed Talks, two sessions
12:45 - 2:00pm	Lunch
2:30 - 3:25pm	Ling Long, LSU *Hypergeometric supercongruences*
3:30 - 4:15 pm	Professional Development Panel
4:20 - 5:10pm	Preston Wake, IAS/MSU *Eisenstein congruences and a Bloch-Kato conjecture in tame families*
5:20 - 6:25pm	Contributed Talks, two sessions

SUNDAY:

9:00 - 10:05am	Contributed Talks, two sessions
10:10 - 11:05am	Brian Smithling, Johns Hopkins University *On Shimura varieties for unitary groups*
11:05 - 11:35am	Coffee/Discussion Time
11:40 - 12:30pm	Arul Shankar, Toronto *Families of elliptic curves ordered by conductor*
12:30pm	Lunch and departure

Schedule of Contributed Talks

Saturday	251 Malott	253 Malott
11:40-12:00	A. Ray	K. Huang
12:02-12:22	A. Haensch	L. Lim
12:25-12:45	X. Xiao	L. Yang

Saturday	251 Malott	253 Malott	203 Malott
5:20-5:40	V. Thatte	Y. Jo	R. Lemke Oliver
5:42-6:02	D. Chow	M. Emory	M. Friedrichsen
6:05-6:25	S. Hong	M. Pandey	C. McWhorter

Sunday	251 Malott	253 Malott
9:00-9:20	C. Springer	S. Yi
9:22-9:42	S. Garai	Q. Shen
9:45-10:05	F. Solomon	C. Zhou

Abstracts of Invited Talks

Abbey Bourdon, Wake Forest University: *Isolated Points on Modular Curves*:

Abstract: We say a closed point on a curve C is isolated if it does not belong to an infinite family of effective divisors of degree d parametrized by the projective line or a positive rank abelian subvariety of the Jacobian of C. In this talk, we will study the image of isolated points under morphisms, giving conditions under which the image of an isolated point remains isolated. We will explore applications of this result to the case where C is the modular curve $X_1(N)$. This joint work with Ozlem Ejder, Yuan Liu, Frances Odumodu, and Bianca Viray extends recent results of the same authors concerning sporadic points on modular curves.

Ling Long, Louisiana State University: *Hypergeometric Supercongruences*

Abstract: In this talk, we will discuss supercongruences occurred to truncated hypergeometric series originated in the work and conjectures of Beukers and Coster. These congruences can be viewed as p-adic analogues of Hecke recursions satisfied by classical modular forms. We will present the related backgrounds under the hypergeometric umbrella and discuss approaches to supercongruences including a p-adic perturbation method proposed in a joint work with Ravi Ramakrishna. The talk will be concluded by applications and new open conjectures based on the motivic setting.

Aaron Pollack, Duke University: *Modular forms on G_2*

Abstract: Classical modular forms are very special automorphic functions for the group $GL(2)$, and similarly holomorphic Siegel modular forms are very special automorphic functions for the group $GSp(2n)$. It turns out that the split exceptional group G_2, and certain forms of the other exceptional groups, possess a similar very special class of automorphic functions. These are called the 'modular forms', and their study was initiated by Gross-Wallach and Gan-Gross-Savin. I will define these modular forms on G_2 and explain what is known about them.

Arul Shankar, University of Toronto: *Families of elliptic curves ordered by conductor*

Abstract: Conjectures on the statistics of elliptic curves are usually formulated with the assumption that the curves in question are ordered by their conductors. However, when proving results on the statistics of elliptic curves, the curves are usually ordered by (naive) height. There are two reasons for doing so: first, it is difficult to rule out the possibility that there are many elliptic curves with small discriminant but large height. Second, it is difficult to rule out the possibility that there are many elliptic curves with large discriminant but small conductor. In this talk, we will focus on the second question, and prove some partial results bounding the number of elliptic curves whose discriminants are much larger than their heights. As consequences, we will construct positive proportion families of elliptic curves, and determine their asymptotics when they are ordered by conductor. We will also prove that the average size of their 2-Selmer groups is 3. This is joint work with Ananth Shankar and Xiaoheng Wang.

Brian Smithling, Johns Hopkins University: *On Shimura varieties for unitary groups*

Abstract: Shimura varieties attached to unitary similitude groups are a well-studied class of PEL Shimura varieties (i.e., varieties admitting a moduli description in terms of abelian varieties endowed with a polarization, endomorphisms, and a level structure). There are also natural Shimura varieties attached to (honest) unitary groups; these lack a moduli interpretation, but they have other advantages (e.g., they give rise to interesting cycles of the sort that appear in the arithmetic Gan-Gross-Prasad conjecture). I will describe some variant Shimura varieties which enjoy good properties from both of these classes. This is joint work with M. Rapoport and W. Zhang.

Preston Wake, IAS & Michigan State University: *Eisenstein congruences and a Bloch-Kato conjecture in tame families*

Abstract: A fact made famous by Mazur is that the Galois representation associated to the modular curve $X_0(11)$ (which is an elliptic curve) is reducible modulo 5. Less famously, the representation is also reducible modulo 25. I'll talk about this extra reducibility and what it has to do with the Bloch-Kato conjecture. This is joint work with Akshay Venkatesh.

Abstracts of Contributed Talks

Dylon Chow, University of illinois at Chicago: *Integral Points on the Wonderful Compactification by Height*
Abstract: I will discuss some recent results on the distribution of integral points on wonderful compactifications of semisimple groups.

Melissa Emory, University of Toronto: *On the global Gan-Gross-Prasad conjecture*
Abstract: In this talk, we discuss work which formulates a Gan-Gross-Prasad conjecture for general spin groups. That is, we formulate a conjecture on a relation between periods of certain automorphic forms on $GSpin(n+1)$ and $GSpin(n)$ along the diagonal subgroup $GSpin(n)$ and some L-values. To support the conjecture, we show the conjecture holds for $n = 2$ and 3 and for certain cases for $n = 4$.

Solomon Fikreab, CUNY Graduate Center: *Subgroup growth zeta functions and Hecke algebras*
Abstract: The study of subgroup growth zeta functions is a relatively young research area. In my thesis, I consider nilpotent groups and I attempt to generalize the notion of the cotype zeta function of an integer lattice to finitely generated nilpotent groups. This helps in determining the distribution of subgroups of finite index and provides more refined invariants in the analytic number theory of nilpotent groups. Similar attempt on algebraic groups leads to a rederivation of zeta functions of classical algebraic groups using Hecke algebras. A connection with Cohen-Lenstra heuristics will also be discussed.

Matthew Friedrichsen, Tufts University: *Comparing the Density of D_4 and S_4 Extensions over Number Fields*
Abstract: When ordered by their discriminant, about 83% of quartic extensions of $\mathbb{Q}$ have associated Galois group S_4, while only 17% have Galois group D_4. We study this disparity over a general number field K by studying the ratio between the number of D_4 and S_4 quartic extensions of K. Over quadratic fields, we show that this ratio can be biased arbitrarily close to 100% of quartic extensions being D_4. Further, we show that in some sense, a bias towards more D_4 quartic extensions is typical of quadratic number fields, significantly contrasting with the case over $\mathbb{Q}$. This is joint work with Daniel Keliher.

Sumita Garai, Pennsylvania State University: *Indices of Endomorphism Ring of a Finite Drinfeld Module over $\mathbb{F}_q[T]$*
Abstract: Let $A = \mathbb{F}_q[T]$ be the polynomial ring over $\mathbb{F}_q$ and F be the field of fractions of A. Let ϕ be a Drinfeld A-module of rank r over F. For all but finitely many primes $\mathfrak{p} \lhd A$, one can reduce ϕ modulo $\mathfrak{p}$ to obtain a Drinfeld A-module $\phi \otimes \mathbb{F}_\mathfrak{p}$ of rank r over $\mathbb{F}_\mathfrak{p} = A/\mathfrak{p}$. It is known that the endomorphism ring $\mathcal{E}_\mathfrak{p} = \mathrm{End}_{\mathbb{F}_\mathfrak{p}}(\phi \otimes \mathbb{F}_\mathfrak{p})$ is an order in an imaginary field extension K of F of degree r. Let $\mathcal{O}_\mathfrak{p}$ be the integral closure of A in K, and let $\pi_\mathfrak{p} \in \mathcal{E}_\mathfrak{p}$ be the Frobenius endomorphism of $\phi \otimes \mathbb{F}_\mathfrak{p}$. Then we have the inclusion of orders $A[\pi_\mathfrak{p}] \subset \mathcal{E}_\mathfrak{p} \subset \mathcal{O}_\mathfrak{p}$ in K. From which we can write $\mathcal{E}_\mathfrak{p}/A[\pi_\mathfrak{p}] \cong A/\mathfrak{b}_1 \times A/\mathfrak{b}_2 \times \cdots \times A/\mathfrak{b}_{r-1}$. In this talk we explore the significance of the indices

$\mathfrak{b}_1, \mathfrak{b}_2, \cdots, \mathfrak{b}_{r-1}$.

Anna Haensch: Duquesne University: *Spinor Regular Ternary Quadratic Forms*
Abstract: It has long been known that integral quadratic forms fail to satisfy a local-global principle. That is, representation of an integer locally at $\mathbb{Z}_p$ for every prime p does not guarantee a global representation over the integers. The failure of the local-global principle is particularly interesting when the form is in three variables. In this talk we explore the underlying structure of such forms and determine for which forms the local-global principle holds, and for which forms the local-global principle almost holds (and why it eventually fails!).

Serin Hong, University of Michigan: *Classification of quotient bundles on the Fargues-Fontaine curve*
Abstract: We completely classify all quotient bundles of a given vector bundle on the Fargues-Fontaine curve. As consequences, we have two additional classification results: a complete classification of all vector bundles that are generated by a fixed number of global sections and a nearly complete classification of subbundles of a given vector bundle. For the proof, we combine the dimension counting argument for moduli of bundle maps developed in [BFH+17] with a series of reduction arguments based on some reinterpretation of the classifying conditions.

Keping Huang, University of Rochester: *Uniform Bounds for Periods of Endomorphisms of Varieties*
Abstract: Suppose X is a variety defined over a finite extension K of $\mathbb{Q}_p$ and which admits a model $\mathcal{X}$ defined over the ring of integers R of K. Suppose $f : X \to X$ is an endomorphism of X defined over K and f can be extended to an endomorphism of $\mathcal{X}$ defined over R. We prove an upper bound for the primitive period of periodic points defined over R.

Yeongseong Jo, University of Iowa: *Rankin-Selberg L-functions via Good sections*
Abstract: In 1990's Bump and Ginzburg establish the integral representation yielding symmetric square L-functions for $GL(n)$ and the twisted version is recently constructed by Takeda. Unfortunately the local functional equation involves intertwining operator opposed to Fourier transform appearing in the well-known Rankin-Selberg integrals for $GL(n) \times GL(n)$ by Jacquet, Piatetski-Shapiro, and Shalika. In this talk, we investigate the modified local Rankin-Selberg integral to incorporate intertwining operator at finite ramifed places. In order to define analogous L-function, we adopt the notion of "Good Sections" introduced by Piatetski-Shapiro and Rallis. The talk will be concluded by describing how this framework is relevant to computing local symmetric square L-functions for $GL(n)$.

Li-Mei Lim, Boston University: *Zeroes of L-functions Associated to Half-Integral Weight Cusp Forms*
Abstract: While automorphic L-functions are expected to satisfy a Riemann hypothesis, L-functions associated to half-integral weight cusp forms in fact do not. In this talk, we'll discuss ongoing work exploring the distribution and behavior of the zeroes of these half-integral weight L-functions.

Caleb McWhorter, Syracuse University: *Torsion of Elliptic Curves over Nonic Galois Fields*

While progress in understanding the ranks of elliptic curves is making slow progress, there has been a surge in the classification of torsion subgroups of elliptic curves over number fields. The purpose of this talk will be to overview some of this recent progress. Then the progress in the classification where K is a nonic Galois field will be discussed.

Robert Lemke Oliver, Tufts University: *Inductive methods for counting number fields*

Abstract: Number fields tautologically come in two flavors: those that contain nontrivial subfields and those that don't. For example, degree n fields whose Galois closure has Galois group S_n or A_n do not admit interesting subfields, while for example D_4-quartic extensions do (and for this reason, they arise with positive density among all quartic fields!). Much work has been focused on problems related to counting families of fields not admitting subfields, but in this talk we instead focus on fields that do admit subfields. In particular, we present a philosophy that makes clear how to think about such problems and admits new counting results, with a particular emphasis on how this philosophy may be used to understand questions related to the class groups of such fields. This is joint work with Jiuya Wang and Melanie Matchett Wood.

Mayank Pandey, Caltech: *On the L^1 norm of an exponential sum with the divisor function*

Abstract: In general for a sequence of complex numbers $a(n)$, the L^1 norm of the trignometric polynomial $\sum_{n\leq X} a(n)e(nt)$ for large X gives some information about how evenly distributed the sequence is in arithmetic progressions. In this talk, we will focus on the case in which $a(n)$ is the divisor function. We shall present a new result in which we obtain an asymptotic formula for the L^1 norm of this exponential sum, improving an earlier result of Goldston and the author. The proof uses Kloosterman's refinement of the circle method along with Voronoi summation.

Anwesh Ray, Cornell University: *Lifts of Mod p Reducible Galois Representations*

Abstract: A Galois representation is (from the perspective and scope of this talk) a continuous p-adic representation of the absolute Galois group of the field of rational numbers. Eigenforms and motives are natural sources for Galois representations, such Galois representations are of central importance in number theory and share some key abstract properties. Since Wiles et al, many sophisticated new techniques have been implemented to show in great generality that Galois representations with suitable properties can be shown to arise from modular/automorphic forms. Serre's Modularity Lifting conjecture, which is now the celebrated theorem of Khare and Wintenberger, predicts when two-dimensional irreducible mod p representation may lift to a modular one. Hamblen and Ramakrishna generalize Khare and Wintenberger's result to two dimensional reducible (and non-semisimple) mod p Galois representations (without being able to optimize the level of the lift). Such mod p representations are abundant and arise naturally from class group data, this is what makes their deformation theory so fascinating. I will outline some generalizations and complements of their result and some potential future directions.

Qibin Shen, University of Rochester: *Linear Relations of v-adic Multi Zeta Values over* $\mathbb{F}_q$

Abstract: We study the linear relations of v-adic MZVs over $\mathbb{F}_q$ and find some families of relations, and we also conjecture that this family generates all linear relations of v-adic MZVs.

Caleb Springer, Pennsylvania State University: *Computing the endomorphism ring of an ordinary abelian surface over a finite field*

Abstract: The endomorphism ring of an ordinary abelian variety over a finite field is isomorphic to an order in a CM field. Knowing the endomorphism ring of a given abelian variety is useful for various problems, such as understanding isogeny graphs and computing class polynomials. In this talk, we present a subexponential algorithm for computing the endomorphism ring by utilizing class groups.

Vaidehee Thatte, Binghamton University: *Ramification Theory for Defect Extensions*

Abstract: Classical ramification theory deals with extensions of complete discrete valuation rings with perfect residue fields. We would like to study arbitrary valuation rings with possibly imperfect residue fields and possibly non-discrete valuations of rank ≥ 1, since many interesting complications arise for such rings. In positive residue characteristic p, the "defect" could be non-trivial (i.e. we can have a non-trivial extension, such that there is no extension of the residue field or the value group). In this talk, we will discuss ramification theory for defect extensions of degree p. Particularly, a generalization of the classical Swan conductor, followed by a generalization and further refinement of Kato's refined Swan conductor map.

Xiao Xiao, Utica College: *Automorphism group schemes at finite level of F-cyclic F-crystals*

Abstract: Let M be an F-crystal over an algebraically closed field of positive characteristic. For every integer $m \geq 1$, let $\gamma_M(m)$ be the dimension of the automorphism group scheme $\mathrm{Aut}_m(M)$ of M at finite level m. In 2012, Gabber and Vasiu proved that $0 \leq \gamma_M(1) < \gamma_M(2) < \cdots < \gamma_M(n_M) = \gamma_M(n_M+1) = \cdots$ where n_M is the isomorphism number of M, and that $\gamma_M(m+1) - \gamma_M(m) \leq \gamma_M(m) - \gamma_M(m-1)$ for all $m \geq 1$ if M is a Dieudonné module over k. We generalize the same result to arbitrary F-crystals in 2014. Questions have been asked whether $\gamma_M(m+1) - \gamma_M(m) < \gamma_M(m) - \gamma_M(m-1)$ for all $1 \leq m \leq n_M$ for any F-crystal M. In this talk, we will discuss a combinatorial formula that calculates $\gamma_M(m)$ for a certain family of F-crystals called F-cyclic F-crystals. This formula allows to give a negative answer to the aforementioned question in general but a positive answer to some family of Dieudonné modules.

Liyang Yang, Caltech: *Modulus Distribution of Local Hecke Traces for $GL(n) : n \leq 4$*

Abstract: Let π be a cuspidal representation of $GL(n)$ over $\mathbb{Q}$, where $n \leq 4$. Let q be an integer. In this talk, we show that there are infinitely many primes $p \equiv \pm 1 \mod q$ such that the local Hecke trace $a_\pi(p)$ lies in the unit disk, i.e., $|a_\pi(p)| < 1$. (Note that this implies π_p is tempered when $n = 3$.) Then a natural problem is to find the least possible prime p such that π_p is unramified and $|a_\pi(p)| < 1$. We give an upper bound for such a prime in

terms of q and the analytic conductor of π. This is a Linnik-type problem on modulus of local Hecke trace.

Shaoyun Yi, University of Oklahoma: *Klingen p^2 vectors for $GSp(4)$*

Abstract: In 2007, Roberts and Schmidt had a satisfactory local new- and oldform theory for $GSp(4)$ with trivial central character, in which they considered the vectors fixed by the paramodular groups $K(p^n)$. In this talk, we consider the space of vectors fixed by the Klingen subgroup of level p^2. We determine the dimensions of the spaces of these invariant vectors for all irreducible, admissible representations of $GSp(4)$ over a p-adic field. The results can be applied to determine the dimension formula for Siegel modular forms of degree 2 with Klingen level 4, which is an ongoing project with Ralf Schmidt.

Changwei Zhou, Binghamton University: *Effective upper bound of analytic torsion under Arakelov metric*

Abstract: Given a choice of metric on the Riemann surface, the regularized determinant of Laplacian (analytic torsion) is defined via the complex power of elliptic operators: $\det(\Delta) = \exp(-\zeta'(0))$. In this paper we gave an asymptotic effective estimate of analytic torsion under Arakelov metric. In particular, after taking the logarithm it is asymptotically upper bounded by g for $g, gt; 1$. The construction of a cohomology theory for arithmetic surfaces in Arakelov theory has long been an open problem. In particular, it is not known if $h^1(X, L) \geq 0$. We view this as an indirect piece of evidence that if such a cohomology theory exists, the h^1 term may be effectively estimated.