Special Workshop for Teachers
May 11, 2019


Peg Smith
Professor Emerita
University of Pittsburgh
Agenda

• Consider the value of classroom discussions

• Compare goals and tasks and consider why they matter

• Discuss a model for orchestrating productive discussions based on students’ work

• Investigate challenges associated with specific practices and how to address

• Consider the implications for your own classroom
Are Discussions Important?
Importance of Discussions

Discussions provide the opportunity for students to:

• Share ideas and clarify understandings

• Develop convincing arguments regarding why and how things work

• Develop a language for expressing mathematical ideas

• Learn to see things for other people’s perspective
A discourse-based classroom affords stronger access for every student--those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach to offer but can offer feedback to others.

Creating Opportunities for Productive Discourse

Ensuring that students have the opportunity to reason mathematically is one of the most difficult challenges that teachers face. A key component is creating a classroom in which discourse is encouraged and leads to better understanding. *Productive discourse is not an accident*, nor can it be accomplished by a teacher working on the fly, hoping for a serendipitous student exchange that contains meaningful mathematical ideas.

Frederick Dillon
Math Teacher for 35 years
Presidential Award Winner
Former NCTM Board Member
Why Orchestrating Discussions is Hard

• Most teachers did not learn mathematics by engaging in discussions

• Discussions reduce teachers’ perceived level of control – you are turning your classroom over to a group of 5-18 year olds!

• Students don’t all solve a task the same way yet the teacher needs to move the entire class toward the same mathematical goal.
Reflection

What is the greatest challenge you face in orchestrating productive classroom discussions?
“The 5 Practices”

- Provides a model for developing and sustaining a discourse-rich math classroom
- Helps teachers balance attention to mathematics and to students
- Supports an equitable mathematics classroom

5 Practices for Orchestrating Productive Mathematics Discussions

Before engaging in the 5 practices, you first need to engage in the “Zero Practice.”
Setting Goals involves specifying what you want students to learn about mathematics as a result of engaging in a particular lesson.

Selecting a Task involves identifying a high-level task that aligns with your goals and provides all students with access.
Anticipating involves carefully considering

- the strategies students are likely to use to approach or solve a challenging mathematical task;
- how to respond to the work that students are likely to produce; and
- which student strategies are likely to be most useful in addressing the mathematics to be learned.
Monitoring involves

• listening in on what students are saying and observing what they are doing;
• asking questions to assess what students understand and to advance them towards the goals of the lesson; and
• keeping track of the approaches that they are using.
Selecting involves

- determining what strategies—and what mathematics—will be the focus of the whole class discussion;
- choosing particular students to present because of the mathematics available in their responses; and
- making sure that over time all students have the opportunity to be seen as authors of mathematical ideas.
Sequencing involves

- purposefully ordering the solutions that will be presented;
- making the mathematics accessible to all students; and
- building a mathematically coherent story line.
5 Practices for Orchestrating Productive Mathematics Discussions

Connecting involves

- asking questions that focus on mathematical meaning;
- asking questions that link different strategies and representations; and
- making sure all students are making sense of the ideas.
Unpack each practice into key components

Explore ways to address challenges associated with each practice

Unpack each practice into key components


Explore ways to address challenges associated with each practice

Authentic video examples

So What’s New?
Meet Jennifer Mossotti

Jennifer Mossotti has been working in the Syracuse City School District in Syracuse, NY since she started teaching 13 years ago.

She is certified to teach mathematics Grades 7–12 as well as students with disabilities in Grades 7–12.

For the past three years she has been teaching mathematics at HW Smith Pre-K–8 School.
5 Practices for Orchestrating Productive Mathematics Discussions

0. Setting Goals
   Selecting a Task

1. Anticipating

2. Monitoring

3. Selecting

4. Sequencing

5. Connecting
Key Questions that Support Connecting Student Solutions

<table>
<thead>
<tr>
<th>What it Takes</th>
<th>Key Questions</th>
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<tbody>
<tr>
<td>Specifying the learning goal</td>
<td>Does the goal focus on what students will learn about mathematics (as opposed to what they will do)?</td>
</tr>
<tr>
<td>Identifying a high-level task that aligns with the goal</td>
<td>Does your task provide students with the opportunity to think, reason, and problem solve?</td>
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<td>Are there resources that I can provide students that will ensure that all students access the task?</td>
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<td>What will I take as evidence that students have met the goal through their work on this task?</td>
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Comparing Goals

Goal A
Students will learn the formula for finding the slope of a line given two points.

Goal B
Students will understand that the rate of change can be seen as the ratio of the change in the $y$-variable, as the rate of change in the $x$-variable, as the rate expressed with the words “for each, per, for every” in a verbal description, or as the coefficient of $x$ in the equation $y = mx + b$.

Goal C
Students will be able to (SWBAT) find the slope of a line given two points.

How are they the same and how are they different?
As a result of engaging in the lesson she wanted her students to understand that

1. the rate of change can be seen as the ratio of the change in the $y$-variable compared to the change in the $x$-variable, as the rate expressed with the words “for each, per, for every” in a verbal description, or as the coefficient of $x$ in the equation $y = mx + b$.

2. some functions do not “start” at zero. That is, the point $(0,0)$ is not a solution for all linear functions.

3. the $y$-intercept can be understood as the initial value of a linear function in a real-world context.

Goal 1: Explore rate of change as price per ticket

Goal 2: All functions do not “start” at zero

Goal 3: Y-intercept can be seen as initial value of function in context
Comparing Three Tasks

Compare Tasks A, B, and C.

• How are the three tasks the same and how are they different?
• What could you learn about students’ thinking from each of these tasks?
Task A

For the problems below, find the slope of the line between each of the two given points. Write your answer in simplest form. To do this without graphing the points.

1) \((1,6)\) and \((3,8)\) = ________________  
2) \((2,5)\) and \((4,7)\) = ________________

3) \((5,6)\) and \((7,10)\) = ________________  
4) \((0,5)\) and \((2,6)\) = ________________

5) \((5,5)\) and \((8,8)\) = ________________  
6) \((2,4)\) and \((5,7)\) = ________________

7) \((1,9)\) and \((2,11)\) = ________________  
8) \((7,5)\) and \((9,9)\) = ________________

9) \((11,6)\) and \((13,9)\) = ________________  
10) \((0,0)\) and \((3,5)\) = ________________
Task B

1. **Vocabulary** Copy and complete: The slope of a nonvertical line is the ratio of the ___ to the ___ between any two points on the line.

2. **Vocabulary** What is the slope of a vertical line?

**Finding Slope** Find the slope of the line.

3. 

4. 

5. 

6. 

7. 

8. 

**Drawing Lines** Draw the graph of the line that passes through the points. Then find the slope of the line.

9. (3, 4), (5, 6) 

10. (2, 5), (5, -2) 

11. (-1, -3), (3, -4)
You are going to Kentucky State Fair in August. You are trying to figure out how much you should plan to spend. The graph shows how much three different people spent after going through the main gate and then buying their ride tickets. Every ride ticket is the same price.

1. After entering the fair, you decide to buy 4 ride tickets. What will be your total cost for attending the fair? How do you know?

2. Describe how the cost increases as you buy more tickets. Be specific.

3. After entering the fair, you decide you want to go on a lot of rides. What will be the total cost for attending the fair and then purchasing 15 ride tickets?

4. Write a description, in words or numbers and symbols that can be used to find the total cost after entering the fair and purchasing any number of tickets.

5. How does the ticket price appear in your description or expression?

6. How does the ticket price appear in the graph?
# Mathematical Task Analysis Guide

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<th>Lower-Level Demands</th>
<th>Higher-Level Demands</th>
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<tr>
<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures with Connections Tasks</strong></td>
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<tr>
<td>Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.</td>
<td>Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.</td>
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<td>Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</td>
<td>Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</td>
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<td>Are not ambiguous — such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.</td>
<td>Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.</td>
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<td>Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
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<th>Procedures Without Connections Tasks</th>
<th>Doing Mathematics Tasks</th>
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<td>Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.</td>
<td>Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</td>
</tr>
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<td>Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>Require students to explore and understand the nature of mathematical concepts, processes, or relationships.</td>
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<tr>
<td>Have no connection to the concepts or meaning that underlie the procedure being used.</td>
<td>Demand self-monitoring or self-regulation of one’s own cognitive processes.</td>
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<td>Are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.</td>
</tr>
<tr>
<td>Require no explanations, or explanations that focus solely on describing the procedure that was used.</td>
<td>Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.</td>
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You are going to Kentucky State Fair in August. You are trying to figure out how much you should plan to spend. The graph shows how much three different people spent after going through the main gate and then buying their ride tickets. Every ride ticket is the same price.

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4. Write a description, in words or numbers and symbols that can be used to find the total cost after entering the fair and purchasing any number of tickets.

5. How does the ticket price appear in your description or expression?

6. How does the ticket price appear in the graph?
Why State Fair is a Good Task

- Relatable context for Syracuse students
- $y$-intercept has meaning in context
- Asks students to generalize (4)
- Relates the rate of change to the context, graph, and general description
- Includes extension questions
Sizing Up a Task

• Are there multiple ways to enter the task and to show competence?
• Does the task require students to provide a justification or explanation?
• Does the task provide the opportunity to use and make connections between different representations of a mathematical idea?
• Does the task provide the opportunity to look for patterns, make conjectures, and/or form generalizations?

So Why All the Fuss About Goals and Tasks?

• A clear goal and a high-level task are necessary conditions for a productive discussion.
  – You must be clear on what it is you want students to learn – “if you don’t know where you are going you could end up someplace else” (Yogi Berra)
  – You have to give students something to do that is worth talking about!
If we want students to develop the capacity to think, reason, and problem solve then we need to \textit{start} with high-level, cognitively complex tasks.

The level and kind of thinking in which students engage determines what they will learn.

Low-level tasks are rarely implemented in ways that result in high-level thinking and reasoning. In general, the potential of the task sets the ceiling for implementation—that is a task almost never increases in cognitive demand during implementation. This finding, robust in its consistency across several studies, suggests that high-level instructional tasks are a necessary condition for ambitious mathematics instruction.

The 5 Practices in Practice - Challenges

Challenge 4:
Launching a Task to Ensure Student Access
Launching a Task

As you watch the video consider how the launch provides students access to the task.
Launching the State Fair Task

• Video can be access on page 31
• Analyzing the Work of Teaching 2.1
How did Mrs. Mossotti’s Launch Provide Students with Access to the Task?

• Students had access to context of lesson.
• Students had opportunity to make observations about the graphs without being restricted to specific questions.
• Students were told that any answer is appropriate. You can’t be wrong!
• Students’ initials next to their ideas acknowledged their contributions.
• Teacher had access to students’ initial thinking about context and graph – signaling that they had a basic understanding that would position them to engage with the questions.
Benefits of an Effective Launch

Students are much more likely to be able to get started solving a complex task, thereby enabling the teacher to attend to students’ thinking and plan for a concluding whole-class discussion. This, in turn, increases the chances that all students will be supported to learn significant mathematics as they solve and discuss the task.

Challenge

Launching a Task to Ensure Student Access
Launching a Task to Ensure Student Access

Challenge

Make sure students:
• understand the context
• can access the mathematics
• know the meaning of key words
• have to figure out how to solve the problem for themselves
Reflection

What did you learn from the discussion of this challenge that would help you in orchestrating discussions in your own classroom?
5 Practices for Orchestrating Productive Mathematics Discussions

0. Setting Goals
   Selecting a Task

1. Anticipating

2. Monitoring

3. Selecting

4. Sequencing

5. Connecting
# Key Questions that Support Anticipating Student Responses

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<thead>
<tr>
<th>What it Takes</th>
<th>Key Questions</th>
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<tbody>
<tr>
<td>Getting Inside the Problem</td>
<td>How do you solve the task?</td>
</tr>
<tr>
<td></td>
<td>How might students approach the task?</td>
</tr>
<tr>
<td></td>
<td>What challenges might students face as they solve the task?</td>
</tr>
<tr>
<td>Planning to Respond to Student Thinking</td>
<td>What assessing questions will you ask to draw out student thinking?</td>
</tr>
<tr>
<td></td>
<td>What advancing questions will help you move student thinking forward?</td>
</tr>
<tr>
<td>Planning to Notice Student Thinking</td>
<td>What strategies do you want to be on the lookout for as students work on the task?</td>
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Going to the State Fair

You are going to Kentucky State Fair in August. You are trying to figure out how much you should plan to spend. The graph shows how much three different people spent after going through the main gate and then buying their ride tickets. Every ride ticket is the same price.

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4. Write a description, in words or numbers and symbols that can be used to find the total cost after entering the fair and purchasing any number of tickets.
5. How does the ticket price appear in your description or expression?
6. How does the ticket price appear in the graph?
Mrs. Mossotti’s Anticipated Solutions

A. Make a table with values from the graph

<table>
<thead>
<tr>
<th>Number of Tickets</th>
<th>Total Spent</th>
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<tbody>
<tr>
<td>1</td>
<td>$8.50</td>
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<td>8</td>
<td>$12.00</td>
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</table>

Student creates a table using the information about the three points on the graph.

B. Determine different unit rates for tickets prices

\[
\frac{8.50}{1} = 8.50 \quad \frac{12}{8} = 1.5 \quad \frac{13}{10} = 1.3
\]

Student divides the total spent by the ticket quantity for each point on the graph and comes up with three different “unit rates.”

C. Determine the price per two tickets

Student uses the points (8, 12) and (10, 13) to determine that two tickets have a cost of $1.00.

D. Connect three points with a line to determine entry fee and ticket price.

Student connects the three points on the graph with a line and sees that at the y-axis the value is $8.00 and determines that it must cost $8.00 to enter the fair without buying any tickets. Since it costs $8.50 for 1 ticket, this means it must cost 50¢ per ticket. Student sees that the line rises half a unit on the y-axis for every 1 unit on the x-axis.

E. Determine price per ticket at 50¢

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Student determines that the total amount spent is calculated by taking $8.00 and then repeatedly adding 50¢ depending on the number of tickets purchased.
Planning to Respond
Assessing and Advancing Questions

Assessing Questions
• Are based closely on the work that the student has produced
• Clarify what the student has done and what the student understands about what he or she has done
• Give the teacher information about what the student understands

Advancing Questions
• Use what students have produced as a basis for making progress toward the target goal of the lesson
• Move students beyond their current thinking by pressing students to extend what they know to a new situation
• Press students to think about something they are not currently thinking about

What Assessing and Advancing Questions Would You Ask Students A and B?

**Student A**
Student creates a table using the information about the three points on the graph.

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**Student B**
Student divides the total spent by the ticket quantity for each point on the graph and comes up with three different 'unit rates.'

\[
\frac{8.50}{1} = 8.50 \\
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5 Practices for Orchestrating Productive Mathematics Discussions

0. Setting Goals

1. Anticipating

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# Key Questions that Support Monitoring

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<tr>
<td>Tracking Student Thinking</td>
<td>How will you keep track of students’ responses during the lesson?</td>
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<tr>
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<td>How will you ensure that you check in with all students during the lesson?</td>
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<tr>
<td>Assessing Student Thinking</td>
<td>Are your assessing questions meeting students where they are?</td>
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<td>Are your assessing questions making student thinking visible?</td>
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<tr>
<td>Advancing Student Thinking</td>
<td>Are you advancing questions driven by your lesson goals?</td>
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<td>Are student able to pursue advancing questions on their own?</td>
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<td>Are your advancing questions helping students make progress?</td>
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## Monitoring Chart

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<thead>
<tr>
<th>Strategy</th>
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- Does this mean you can only buy 1 ticket, 8 tickets, or 10 tickets?
- Does this mean that 1 ticket has a cost of $8.50?
- What is a way to determine values that are missing from the table?
- How much would 9 tickets cost? How do you know?
- What if I don’t buy any ride tickets? Would I need to send any money?
- So sometimes tickets have different prices? How do I get to buy the cheap ticket? something seems funny here?
- What do each of these numbers mean?
- 8.5 what? For what?
- Why is one ticket so expensive but 12 tickets so cheap, for each ticket? Are they having a special sale that I don’t know about?
- Is there a way to use the values from each point to find out a single ticket price?
The 5 Practices in Practice - Challenges

Challenge 8:
Trying to Understand What Students are Thinking
Trying to Understand What Students are Thinking

Solution 1
(Daejhor, Razaria, and Mya)

Solution 2
(Serenity)

Analyze the work of these students. For each solution consider:

1. What does the student appear to understand?
2. What seems to be confusing the student?
3. What would YOU do if your students produced these solutions?
In this video clip you will investigate Daejhor, Razaria, and Mya’s work on the State Fair task.

As you watch the clip, consider:

What does Mrs. Mossotti do to determine what students are thinking?
What Are Students Thinking?
Part One

• Video can be accessed on page 77
• Analyzing the Work of Teaching 4.3
What Are They Thinking?
Part Two

When Mrs. Mossotti first visited Serenity and Adnawmy they were confused about how to begin the task. After a brief conversation, the teacher challenged them to estimate the cost of 4 tickets. In this Video Clip, she returns to the pair to see what progress they have made.

As you watch the clip, consider:

What does Mrs. Mossotti do to determine what students are thinking?
What Are They Thinking?
Part Two

• Video can be access on page 85
• Analyzing the Work of Teaching 4.6
What Are They Thinking?

- What does Mrs. Mossotti do to support (or inhibit) her students ability to make progress on the task?
- Why did she walk away from students before ensuring that they had the right answer?
What Mrs. Mossotti did to Understand Student’s Thinking?

• The teacher asked *assessing* questions to determine what students had done and what they had figured out.

• The teacher listened closely to the students’ explanations and pressed for clarity.

• The teacher pressed students to consider their current answer in light of the graphical representation.

• The teacher did not tell students what to do and never indicated whether an answer was correct or incorrect.

• The teacher left the students to continue to work on an *advancing* question on their own.
Challenge

Trying to Understand What Students are Thinking
Challenge

Trying to Understand What Students are Thinking

Addressing the Challenge

• Ask questions
• Be persistent
• Don’t tell students what to do – this is a short term solution that creates dependence
• Anticipate solutions and questions in advance so you are ready for this!
Reflection

What did you learn from the discussion of this challenge that would help you in orchestrating discussions in your own classroom?
5 Practices for Orchestrating Productive Mathematics Discussions

0. Setting Goals
Selecting a Task

1. Anticipating

2. Monitoring

3. Selecting

4. Sequencing

5. Connecting
Key Questions that Support Selecting and Sequencing

<table>
<thead>
<tr>
<th>What it Takes</th>
<th>Key Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying Student Work to Highlight</td>
<td>Which student solution strategies would help you accomplish your mathematical goals for the lesson?</td>
</tr>
<tr>
<td></td>
<td>What challenges did students face in solving the task? Where there any common challenges?</td>
</tr>
<tr>
<td>Purposefully Selecting Individual Presenter</td>
<td>Which students do you want to involve in presenting their work?</td>
</tr>
<tr>
<td></td>
<td>How might selecting particular students promote equitable access to mathematics learning in your classroom?</td>
</tr>
<tr>
<td>Establishing a Coherent Storyline</td>
<td>How can you order student work such that there is a coherent storyline related to the mathematics learning goals?</td>
</tr>
</tbody>
</table>
Mrs. Mossotti’s Monitoring Chart

• Review Mrs. Mossotti’s monitoring chart. What do you notice?

• As noted on the chart, Mrs. Mossotti decided to have two groups present their solutions:
  – Daejhor, Mya, + Razaria – 1st
  – Crispin + Nazier – 2nd

Why do you think she selected these students in this order?
Mrs. Mossotti’s Monitoring Chart

<table>
<thead>
<tr>
<th>Solution Strategy</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
<th>Who and What</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students cannot get started.</td>
<td>- Where on the graph do you estimate 4 tickets would be?</td>
<td>- Is there a way to determine the actual amount spent for 4 tickets besides estimating?</td>
<td>Adamwmm + Serenity - not sure of cost</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Let’s role play, here’s my wallet, and I’m about to enter the state fair… you tell me what happens and when things happen…</td>
<td></td>
<td>2nd visit - take average of 3 rates - get $3.77/ticket</td>
<td></td>
</tr>
<tr>
<td>Student creates a table using the information about the three points on the graph.</td>
<td></td>
<td></td>
<td>No one used a table!</td>
<td></td>
</tr>
<tr>
<td>Number of Tickets</td>
<td>Total Spent</td>
<td>Number of Tickets</td>
<td>Total Spent</td>
<td>Number of Tickets</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------</td>
<td>-------------------</td>
<td>-------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>1</td>
<td>$8.50</td>
<td>8</td>
<td>$12.00</td>
<td>10</td>
</tr>
</tbody>
</table>

- So some of the tickets are at different prices. how do I get to buy the cheap ticket... something seems funny here?
- What do each of these numbers mean?
- 8.5 what? For what?

- So 3 tickets so 3 x $8.50 = $25.50 each ticket? Are they having a special sale that I don't know about?
- Is there a way to use the values from each point to find out a single ticket price?

- $8.50 for 1 so $8.50 x 4 for 4 tix
  - Binti, Mohamed, Daehor, Mya, Razaria
  - Claire + Fadumo

- 2nd visit - Daehor, Mya, Razaria - 1" came back with actual cost for 4 tix - look at graph

- Lots of kids thought 4 tix - $8.50 each so $34 start here!

- Daehor, Mya, Razaria - 1"
The 5 Practices in Practice - Challenges

Challenge 15: Determining how to sequence errors, misconceptions, and/or incomplete solutions
Sequencing Incorrect Work

Mrs. Mossotti invited Daejhor, Razaria, and Mya to present their work to the class. Recall that this group had initially determine that the cost of 4 tickets would be $8.50 (below left). They subsequently drew a line through the three give points (below right).
Sequencing Incorrect Work

As you watch the video clip, consider the following questions:

• Why do you think Mrs. Mossotti decided to start the discussion with an incorrect answer? What might be the benefit in this approach?

• What did Mrs. Mossotti do to involve the entire class in the discussion?
Sequencing Incorrect Work

- Video can be accessed on page 136
- Analyzing the Work of teaching 6.1
Why Did Mrs. Mossotti Started with an Incorrect Solution?

• Lots of students thought 4 tickets would cost $34.00, so getting this on the table for discussion early on would provide an opportunity for students to reconsider their initial thinking if they had not already done so

• Daejhor, Mya and Rozaria eventually determined that 4 tickets cost $10.00 so this provided the opportunity for them to explain why they changed their minds. This validated that it is okay to change your mind.

• The second solution made clear that the cost per ticket was 50¢ and why and the graph they shared shows that the line does not start at the origin. This presentation laid the groundwork for the subsequent discussion and the first step in achieving the lesson goals (see goals 2 and 3).

• The entire class was held accountable for listening to the group’s explanation and making sense of their reasoning.
Challenge
Determining how to sequence errors, misconceptions, and/or incomplete solutions
Challenge
Determining how to sequence errors, misconceptions, and/or incomplete solutions

Addressing the Challenge

• Identify errors or misconceptions that are held by several students or are common in the domain
• If getting beyond the error or misconception is key to accomplishing your lesson goals – plan to address it directly
• Select a student to present who has worked through their initial confusion, or present the error without identifying who produced it … “I noticed some students said 4 tickets cost $34. Who agrees? Disagrees?”
Reflection

What did you learn from the discussion of this challenge that would help you in orchestrating discussions in your own classroom?
Connecting involves

- asking questions that focus on mathematical meaning;
- asking questions that link different strategies and representations; and
- making sure all students are making sense of the ideas.
Key Questions that Support Connecting Student Solutions

<table>
<thead>
<tr>
<th>What it Takes</th>
<th>Key Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connecting Student Work to the Goals of the Lesson</td>
<td>What questions about the student work will make the mathematics being targeted in the lesson visible?</td>
</tr>
<tr>
<td>Connecting Different Solutions to Each Other</td>
<td>What questions will help students make connections between the different solution strategies presented?</td>
</tr>
</tbody>
</table>
Challenge 17: Ensuring key mathematical ideas are made public and remain the focus
Polling the Class

Following the presentation given by Daejhor and Mya, the Mrs. Mossotti polled the class to see what they thought the price per ticket was now.

“I’m going to have you guys pause for a second. . . close your eyes, close them or cover them. If you think the cost of one ticket is $3.77 raise your hand.” After waiting a few seconds, she continued, “Put your hands down. If you think the cost of one ticket is 50¢ raise your hand.” She again waited briefly. “If you think that the cost of one ticket is $8.50 raise your hand. OK, put your hands down.”

Why do you think Mrs. Mossotti did this?
Student Presentation 2
Serenity

As you watch the next video clip, consider Serenity’s explanation of why she now thinks that 1 ticket costs 50¢ and Mrs. Mossotti’s comments and questions.

• What questions does Mrs. Mossotti ask to connect Serenity’s idea to lesson goals?
Second Presentation
Serenity

• Video can be accessed on page 138
• Analyzing the Work of Teaching 6.2
As you watch the final clip, consider Crispin and Nazier’s explanation of how they discovered that each ticket cost 50¢, and listen to Mrs. Mossotti’s comments and questions.

• What does Crispin and Nazier’s solutions add to the discussion?
Crispin and Nazier

- Video can be accessed on page 141
- Analyzing the Work of Teaching 6.3
How Did Mrs. Mossotti Keep the Focus on the Key Ideas?

• What did Mrs. Mossotti do to ensure that the key mathematical ideas she had target during the lesson were made public?

• What did Mrs. Mossotti do to engage all students in the class in thinking about these ideas?
What Did Mrs. Mossotti Do?

• Selected student work that would advance the goals of the lesson
  – all 3 presenters had ultimately concluded that the cost per ticket was 50¢
  – all three presenters showed a y-intercept of $8.00
• Pressed the presenters to explain what they were thinking and why
• Kept the focus on the graph and the context
• Engaged other students in making sense of what the presenter was saying
Challenge

Ensuring key mathematical ideas are made public and remain the focus.
Ensuring key mathematical ideas are made public and remain the focus

Addressing the Challenge

- Select student responses that can be used to surface the ideas that you want to make public
- Ask specific questions that get the ideas on the table for discussion
- Continue to press students to make connections between the work being discussed and the ideas you are trying to help them understand
- Make sure that several different students are contributing to the discussion to ensure that the ideas are voiced in different ways
- Give students time to process the ideas that have been put forth
Reflection

What did you learn from the discussion of this challenge that would help you in orchestrating discussions in your own classroom?
Is Using the 5 Practices Really Worth the Effort?
Simply put, the five practices can equip teachers in supporting students’ work on challenging tasks without lowering the demands of the task. In particular, by anticipating what students are likely to do when solving the task (including not being able to get started) and the questions that can be asked to assess and advance their understanding, the teacher is in a much better position to provide scaffolds that support students’ engagement and learning without taking over the thinking for them.

I think a lot of teachers (myself included) felt successful after seeing kids successfully solve an equation or complete a series of steps to solve a problem. But looking back, the student success was more characteristic of compliance and understanding of the underlying concepts was actually very shallow. Building a foundation based upon a concept prior to teaching the procedure may seem daunting as everyone is rushing to ‘fit the curriculum in,’ but it will always save time in the long run. Even if it feels like a failure the first time, or even if it feels like it’s taking a lot more time than you anticipated, that time is going to be earned back when students have that conceptual understanding...

Jen Mossotti’s Perspective

Some students present challenging behavior, some are harder workers than others, some have language barriers, some have issues at home that are beyond my comprehension, and some are on grade level and many are not. But at the end of the day, the work of the five practices will bring the largest opportunities for all students to show progress from their current level and will outweigh typical stand-and-deliver instruction.

Conclusion

• Discussions are critical if we want students to learn mathematics with understanding.
• The 5 Practices model can help you in planning for and managing discussions.
• Working with colleagues can help you in thinking through the lesson in advance.
• Being aware of the potential challenges can help you prepare for and work through them.
Reflection

What will you do to improve the quality of discussions in your classroom?


thank you