2006-2007		
Discrete Probability ***Instructor: Abra Brisbin (CAM)*** The first few days introduced set theory and combinatorics. After that, they turned to probability distributions, expected value, and independence. In the second half of the course, they worked on conditional probability, Bayes' Theorem, the Monte Carlo method, and Markov chains. Throughout the course, they tackled questions involving applications of probability in biology, medicine, social policy, and everyday life.	**Game Theory** ***Instructor: Jason Anema (Math)*** This course began with a study of matrix games and a proof of the existence of Nash equilibria. Students then studied decision graphs, which included backwards induction, uncertainty and multiple-person decisions, and as an example played "Indian Poker" in class. The next topic was the problem of maximizing utility in auctions with incomplete information. The course concluded with a consideration of voting schemes and coalitions.	**Revisiting Combinatorics** ***Instructor: Jay Schweig (Math)*** Combinatorics was taken in new directions for the students, covering graph and tree enumeration, partitions, compositions, and generating functions. Responding to a request from the students, the second part of the session covered the beginnings of formal logic, including a treatment of sentential logic, a discussion of the completeness and incompleteness theorems, and a full proof of the compactness theorem for sentential logic.
2007-2008		
Combinatorics: Unusual Counting Problems ***Instructor: Gwyneth Whieldon (Math)*** Students started with proofs of interesting Fibonacci identities, and then moved on to more general binomial identities, with the emphasis on using bijective counting arguments rather than induction or other proof methods. Lucas and Gibonacci identities were studied, and more difficult identities that combine Lucas and Fibonacci numbers were studied. Binet's formula using combinatorial and probabilistic arguments was proved. Identities on simple or general continued fractions was studied. Students were introduced to Khinchin's constant and some of its more unusual properties.	**Group Theory** ***Instructor: Jonathan Needleman (Math)*** This course emphasized symmetries of mathematical objects, such as geometric shapes and sets. Basic properties of groups were explored including subgroups, normal subgroups, and quotient groups. Lagrange's Theorem and the first isomorphism theorem were proved, and the Sylow Theorems were stated. For an end of the term group project the students decided to explore symmetries in M.C. Escher's artwork.	**Introduction to Knot Theory** ***Instructor: Victor Kostyuk (Math)*** This session started with basic concepts of knot and link projections, ambient isotopies, and Reidemeister moves, and continued with simple link and knot invariants (e.g., linking number, tricolorability, and crossing number). Alexander and Jones polynomials were introduced, the latter defined in terms of the Kauffman bracket. Students examined the Dawker notation and algebraic tangles, closed braid representations of links, torus and satellite knots. The last few weeks were spent discussing surfaces and the Euler characteristic, leading to a definition of a knot's genus and construction of Seifert surfaces.

2008-2009		
Counting Problems & Generating Functions ***Instructor: Saul Blanco Rodriguez (Math)*** Students looked at the connection between rational generating functions and linear recurrences, and used these to find closed formulas for the Fibonacci and Catalan numbers that were defined recursively as the solution to counting problems. Students explored bijections between objects that are counted by Catalan and Motzkin numbers. Other topics included composition, set and number partitions, their associated generating functions, and Euler's Pentagonal number theorem. Unlike the generating functions connected to linear recurrences, students discovered that the generating functions associated with partitions are an infinite product and not an infinite sum.	**Cardinality** ***Instructor: Matt Noonan (Math)*** This session extended the unit on generating functions and examined how generating functions can lead to a generalized notion of cardinality. Students applied generating functions to the construction of "nonstandard dice." Understanding these dice is closely tied to understanding cyclotomic polynomials, and this became the new theme of the course. The seminar moved on to study constructability of regular polygons by ruler and compass. Finally, after studying some basic number theory in the form of Fermat's Little Theorem, Wilson's Theorem, and Euler's Theorem, the RSA encryption algorithm was introduced.	**Isometries & Symmetries** ***Instructor: Victor Kostyuk (Math)*** Students looked at isometries of the line and plane, and symmetries of figures in the plane. This led to groups of isometries or symmetries. They discussed the axioms a set needs to satisfy in order to be a group, and studied basic examples of groups, their properties, and geometric expression as symmetries or isometries. Injective and surjective functions were covered, as well as homomorphisms and isomorphisms. They explored kernels, normal subgroups, quotient groups, and the first isomorphism theorem. Seminar closed with a discussion of generators, relations, and free groups. Universal properties and commutative diagrams were introduced in this context.
2009-2010		
Group Theory: Rubik's Cube ***Instructor: Jennifer Biermann (Math)*** Students first focused on determining the size of the Rubik's cube group. For this they learned about permutation groups, decompositions of permutations into disjoint cycles, and even and odd permutations. They investigated subgroups of the Rubik's cube group and Lagrange's Theorem. The last part of the unit touched on several subjects as Cayley graphs and quotient groups. The students were not taught how to solve the Rubik's cube but rather, time was spent discussing conjugates and how they could be used to find useful sequences of moves. Most students learned (on their own) how to solve the cube by the end of the unit.	**Paradoxes and Infinity** ***Instructor: Gwyneth Whieldon (Math)*** This seminar began with discussion of several classical math paradoxes, properties of numbers and number systems, and the development of axiomatic mathematics. Students examined sizes of finite and infinite sets and several classic puzzles and paradoxes (e.g., Zeno's paradox, the Banach-Tarski paradox, the Littlewood Ping-Pong ball problem, and other Supertask problems). Other topics included partial fractions and Khinchin's constant, Fibonacci numbers and counting problems, and properties of infinite sums. Students took an historical examination of paradoxes of set theory, and the axiomatic systems of Whitehead and Russell.	**Introduction to Cryptology** ***Instructor: Benjamin Lundell (Math)*** Basic Caesar and Rail Fence ciphers opened the seminar. After introducing modular arithmetic, students generalized to multiplication and, eventually, affine ciphers. Students studied monoalphabetic substitution ciphers using the Keyword cipher, and the cryptoanalysis of ciphers through frequency analysis and letter distributions. They learned ways to make more secure ciphers, which led to the Vigenere cipher and the one-time pad. They studied cryptoanalysis of polyalphabetic ciphers via the Friedman test, public key cryptography, and the Diffe-Hellman Key Exchange Protocol. We ended with a week-long "cipher scavenger hunt," in which students cracked a series of ciphers leading to a prize hidden in the room.

2010-2011		
Counterexamples in Mathematics ***Instructor: Mircea Pitici (Math Education)*** This seminar was based exclusively on analyzing, constructing, and discussing counterexamples in mathematics. We proceeded gradually, starting with counterexamples pertaining to basic functional notions and quickly advancing to counterexamples related to functional properties studied in calculus (continuity, differentiability, Darboux property, integrability). Among many examples we included some of historical importance (e.g., Dirichlet function and its variants, Weierstrass function). Most examples concerned functions of one variable, but toward the end we also studied counterexamples in functions of two variables. The initial intent was to include all branches of mathematics, but we decided to stay just within calculus for the whole seminar. However, as a final project, one student studied counterexamples in number theory.	**Probability** ***Instructor: Tilo Nguyen (CAM)*** The first segment of this seminar was an introduction to probability, starting with some refresher combinatorics problems. Students then learned how to use combinatorics and Venn diagram to calculate probability. We talked about the meaning of independent or mutually exclusive events. We discussed conditional probability and Bayes' Theorem, using disease testing as an example. We studied popular discrete distributions (e.g., Bernoulli, binomial, negative binomial, geometric, and Poisson distributions). We studied Markov chains and briefly discussed the use of Markov chains in real life applications (e.g., Google searches). The seminar ended with solving fun and famous probability problems (e.g., Buffon's needle).	**Numerical Analysis** ***Instructor: Amy Cochran (CAM)*** The seminar began with preliminary topics: computer arithmetic, error, logic, and basic programming using graphing calculators. The bulk of the seminar was focused on linear and nonlinear systems of equations. For linear systems, the students examined solution techniques (e.g., Gaussian Elimination, Backward/Forward Substitution, and Jacobi method), and learned about matrix and vector norms, singular value decomposition, and LU decomposition. For nonlinear systems, root-finding and minimization techniques were studied, including Newton-Raphson, bisection, golden search, and steepest descent methods. Numerical calculus was also briefly studied. Topics included Newton-Cotes formulas, Gaussian, and Monte Carlo integration.
2011-2012		
Special Curves ***Instructor: Mircea Pitici (Math Education)*** We explored various special curves, with an emphasis on geometric elements; occasionally we also studied the algebraic and trigonometric properties, and pointed out the importance of the curves in applications, for instance in the theory and construction of mechanisms. All along we considered other curves related to a given curve, such as pedal curves and envelopes. We started off with a unified view of conics offered by projective geometry, mentioning several major closure theorems (due to Pascal, Brianchon, Poncelet) and a potpourri of side results. We continued by examining in detail the cycloid, a few particular epicycloids and hypocycloids (cardioid, astroid, deltoid, nephroid), and various spirals. Pressed by time, we mentioned cursorily some of the properties of limaçon, lemniscates, and ovals.	**Axiomatic Development of Probability** ***Instructor: Mark Cerenzia (Math)*** This seminar presented the axiomatic approach to probability theory so that students could learn how mathematical machinery is built and applied. We began with a brisk introduction to relevant set theory in order to state the three axioms of probability (i.e., the definition of a measure space). We then derived typical properties one would expect when computing probabilities and showed how the framework helps us avoid pitfalls that both laymen and professionals often make. This led naturally into other core concepts, such as independence, conditional probability, Bayes' Formula, and random variables along with their important quantities (variance and expectation). Deriving everything formally from the axioms was the main feature and focus of this development.	**Calculus of Variations** ***Instructor: Anoop Grewal (TAM)*** We started off with the historical beginning of the subject with the famous Brachistochrone problem by Johann Bernoulli. The general solution by Euler and Lagrange was derived and discussed next. We discussed many famous applications of calculus of variations in engineering and physics, including geodesics on the plane, cylinder and sphere; Lagrangian formulation of mechanics; and the catenary curve as minimum potential energy solution.

2012-2013		
Complex Numbers and Geometry ***Instructor: Mircea Pitici (Math Education)*** During this seminar we reconstructed much of the school geometry (and more) from a novel perspective: by using complex numbers. We defined special operations with complex numbers (such as the "real product" of two complex numbers and the "complex product" of two complex numbers) which lead to remarkable mathematical expressions for basic geometrical relationships and concepts (such as collinearity, concurrence, area) and introduced alternative systems of coordinates (i.e., baricentric coordinates). With these elements we proved important geometrical results of historical, theoretical, and educational value—some well-known and others little known. This was a capstone seminar during which we integrated elements of different mathematics branches including geometry, algebra, linear algebra, and trigonometry.	**Set Theory and the Foundations of Mathematics** ***Instructor: Iian Smythe (Math)*** We explored how sets can be used to axiomatically build up the whole of mathematics. Beginning with the basic algebra of sets, we built new sets from old ones and defined functions, relations, and numbers in terms of sets. Detours into the worlds of orders, graphs, and other relational structures were made along the way. We established induction on the natural numbers and constructed explicitly the basic arithmetical operations. From here, the integers, rationals, and reals (via Dedekind cuts) were constructed, and their basic properties were discussed, including those used in the foundations of calculus. Lastly, we turned to the issues of cardinality, countable sets, uncountable sets, Cantor's Theorem, and basic cardinal arithmetic. As an epilogue, we briefly discussed other foundational issues, such as the Continuum Hypothesis, Godel's Incompleteness Theorems, Turing machines, and the undecidability of the halting problem.	**Axiomatic Development of Probability** ***Instructor: Mark Cerenzia (Math)*** In slight contrast to the previous year, this seminar focused on applying the axiomatic framework of probability to approach concrete problems and concepts. Notes were distributed to facilitate and hasten the acquisition of this machinery, allowing us to cover paradoxes (notably Simpson's), distributions of random variables, and Markov Chains. The highlight of the seminar was a study of Brownian Motion and its properties, including a computation of distributions of certain of its functionals, computation of probabilities of concrete events (such as the probability it hits zero on a given interval, which can be expressed explicitly with the arccos function), and lastly Marc Kac's derivation of the arcsin distribution of the time Brownian Motion spends in the positive axis.
2013-2014		
The Isoperimetric Problem ***Instructor: Hung Tran (Math)*** This seminar was motivated by Queen Dido's problem, isoperimetric regions on a plane. The first half was devoted to studying the history, practical relevance, geometry, and symmetrization techniques. We also discussed variants of the problem in different contexts. In the second half, several proofs of the main theorem, which asserts that circular balls are minimizers, were derived. Last but not least, we talked about related issues such as compactness arguments, Gromov's magical construction, and bubbles.	**Classical Algebraic Geometry** ***Instructor: Sergio Da Silva (Math)*** This session was devoted to learning classical algebraic geometry (which is the study of solutions to algebraic equations) by focusing more on the motivational aspects of the problems rather than the abstract techniques and framework introduced in the 20th century. The basic concepts of affine and projective varieties and their morphisms were introduced. The first half built up to Bezout's Theorem, while the second portion focused on the resolution of singularities (at least for curves and surfaces). Some classical constructions such as the Segre and Veronese embeddings were discussed, as well as the process of blowing up a point in affine space.	**Reflection Groups** ***Instructor: Balazs Elek (Math)*** First we looked at a couple examples of groups, which we have shown to be reflection groups after investigating how reflections interact with each other in Euclidean space. To motivate the classification of all finite reflection groups, we looked at regular polytopes, and investigated some of their combinatorial properties. Then we looked at root systems associated to finite reflection groups, and proved several theorems in detail with the aim of showing that every such group is generated by a set of simple reflections. We used this knowledge to construct the Coxeter graph of a finite reflection group, and we used Kostant's find the highest root game to give a combinatorial proof of the classification of Coxeter graphs of finite type.

2014-2015		
Introduction to Combinatorics ***Instructor: Sergio Da Silva (Math)*** Combinatorics is the study of finite or countable discrete structures. This session was devoted to learning classical enumerative techniques, which eventually branched off into graph theory. The basic concepts of enumerative combinatorics were taught, including general counting methods, generating functions, recursion relations, the inclusion/exclusion principle, and rook polynomials. A small portion was devoted to algebraic combinatorics, such as Polya enumeration and to game theory. Topics in graph theory included basic definitions, planar graphs, graph coloring, Hamiltonian circuits, and various algorithms. In fact, one of the final projects was a problem from graph theory featuring the Hoffman-Singleton graph and related topics.	**Algebraic Obstructions in Topology** ***Instructor: Aliaksandr (Sasha) Patotski (Math)*** This seminar was intended to be an introduction to algebraic topology, with the emphasis on the algebraic side. We considered some basic topological objects: knots and links, surfaces, and vector fields on them. Having these object in hand, there are plenty of questions you can ask: are given two knots isotopic? Can we classify surfaces up to homeomorphism? Is this surface orientable? Are the two given vector fields homotopic? Can we classify them on a given surface? It turns out that there are rather pretty algebraic constructions helping to answer these questions, and studying them was the main goal of the course. Students learned about polynomial invariants of knots, about the Euler characteristic of a surface, about indices of vector fields, and how these things relate to each other and how they help us to study topological objects.	**Fourier Analysis** ***Instructor: Evan Randles (CAM)*** The seminar focused on Fourier series of periodic functions on the real line. After introducing some basics and history of Fourier series, the seminar was broken into three segments. In the first segment, the students were introduced to uniform convergence, integration, and the exchange of limits. This segment ended with the basic result: Every twice continuously differentiable periodic function has a uniformly convergent Fourier series. In the second segment, the focus turned to applications and included a detailed discussion and solution to the heat equation. The third segment returned to the question of convergence, in which the students learned about pointwise convergence, the Dirichlet kernel, and the Gibbs' phenomenon. The seminar ended with the statement of the celebrated theorem of Carleson.
2015-2016		
Introduction to Number Theory ***Instructor: Sergio Da Silva (Math)*** The study of the integers is a primary focus of number theory. During this seminar the students learned about classical results from the subject, including finding integer solutions to Diophantine equations, the Euler totient function, and the Euclidean algorithm. Two main goals of the seminar were to understand the RSA algorithm that is used for encryption, and to introduce elliptic curves. Fermat's Last Theorem was also discussed. Final projects on elliptic cryptography and Pell's equation were chosen by the students.	**Group Theory Via Interesting Examples** ***Instructor: Aliaksandr (Sasha) Patotski (Math)*** This course was an introduction to group theory, emphasizing examples and applications of group theory to mathematics and real life. The group theoretic topics were rather standard, and included definitions of groups, homomorphisms, group actions, Lagrange's theorem, quotient groups, and Cayley graphs. Abstract notions and theorems were only introduced when motivated by a situation or a problem not directly involving groups. For example, symmetric groups were used to prove that the famous Sam Loyd's puzzle is unsolvable. As another example, the problem of creating bell ringing patterns for churches motivated the study of Cayley graphs.	**Projective Geometry** ***Instructor: Daoji Huang (Math)*** Projective geometry was introduced from the synthetic point of view. The axioms of projective space and duality on the projective plane were discussed, and some basic properties of projective spaces were proved. Then a few simple finite projective spaces were studied. After that, a different point of view was taken, and homogeneous coordinates and projective transformation using some linear algebra were introduced, which was explained along with applications in computer graphics. In the last part of the course, the topics discussed were cross ratio, Desargue's theorem, Pappus' theorem, and geometric constructions of sums and products, with an emphasis on the interplay between geometry and algebra.

2016-2017		
Reflection Groups ***Instructor: Balazs Elek (Math)*** First we looked at a couple examples of groups, which we have shown to be reflection groups after investigating how reflections interact with each other in Euclidean space. To motivate the classification of all finite reflection groups, we looked at regular polytopes, and investigated some of their combinatorial properties. Then we looked at root systems associated to finite reflection groups, and proved several theorems in detail with the aim of showing that every such group is generated by a set of simple reflections. We used this knowledge to construct the Coxeter graph of a finite reflection group, and we used Kostant's find the highest root game to give a combinatorial proof of the classification of Coxeter graphs of finite type.	**Continued Fractions** ***Instructor: Gautam Gopal Krishnan (Math)*** This seminar was an introduction to working with real numbers using continued fractions. We looked at how Euclid's algorithm to compute the greatest common divisor of two integers can be used to compute the continued fraction of a rational number. Using this, we then studied how to best approximate irrational numbers by rational numbers. This led to a discussion of different notions of "best" approximations. A geometric approach to think about continued fractions and approximations was also considered. We then discussed applications of continued fractions, and studied Diophantine equations using continued fractions.	**Topological Data Analysis** ***Instructor: Amin Saied (Math)*** In this module students learned how to apply ideas from topology to analyze so-called "big data." Examples included the following: • Using graph theory to simulate the dynamics of the Internet, eventually leading to Google's famous Page Rank algorithm; and • Using "persistent homology" to investigate the manifold hypothesis, that is, the idea that high-dimensional data sets appearing in nature tend to conform to low dimensional manifolds. https://aminsaied.github.io/topology-and-data-analysis/
2017-2018		
Understanding Computation ***Instructor: Daoji Huang(Math)*** In this seminar students studied computations from two different perspectives. The first was "abstract machines," namely, automata theory. Students were introduced to deterministic and non-deterministic finite automata and it was shown that non-deterministic finite automata can be determinized. Students then were introduced to Turing machines and shown that they are more powerful than finite automata. The halting problem was also discussed. In the end, students briefly studied the second perspective, which is an "abstract program," namely, lambda calculus.	**Reflection Groups** ***Instructor: Balazs Elek (Math)*** First we looked at a couple examples of groups, which we have shown to be reflection groups after investigating how reflections interact with each other in Euclidean space. To motivate the classification of all finite reflection groups, we looked at regular polytopes, and investigated some of their combinatorial properties. Then we looked at root systems associated to finite reflection groups, and discussed several theorems with lots of hands-on computations and examples.	**Numerical Methods** ***Instructor: Matthew Hin (CAM)*** The seminar was an introduction to basic numerical methods and programming principles using Python. After introducing how computers represent numbers and how errors can propagate, the seminar was divided into four segments. 1. Students were introduced to Gaussian elimination and its uses in solving linear systems. 2. Students were introduced to bisection methods and Newton-Raphson methods and their uses in solving nonlinear equations. 3. Students learned about Lagrange interpolation and its application to Newton-Cotes integration. 4. Students explored the forward, improved, and backward Euler methods for ordinary differential equations.