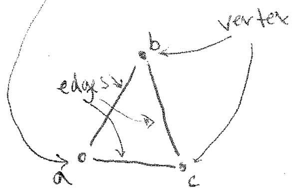
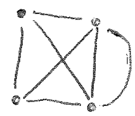
INTRODUCTION TO GRAPHS February 2, 2013

GRAPH

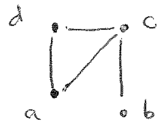




NOT A GRAPH

Edge - a connection between a pair of vertices.
Graph - a collection of vertices V and edges E.

Example:
$V = \{a, b, c, d\}$, $E = \{(a, c), (a, d), (b, c), (c, d)\}$



neighbors of a vertex are those connected by an edge. Ex: neighbors of c are d, b, a. neighbors of b are c.

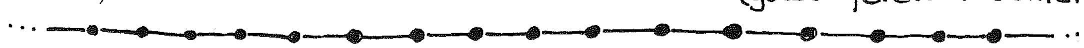
Is this a graph?

a)

yes! graphs don't have to be connected.

b)

(goes on forever to both sides)



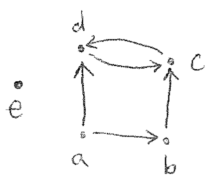
yes! graphs don't have to be finite.

Directed graph - a graph whose edges are directed (i.e. have a beginning and an end).

Example:

edge starts at a and ends at b

$V = \{a, b, c, d, e\}$, $E = \{(a, b), (b, c), (c, d), (d, c), (a, d)\}$ order matters in these pairs!



This is one drawing of the graph.

Applications

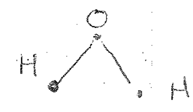
- Friendships (V=individuals, E=pairs of friends)
  Is this a directed graph? no

If I'm friends with you, then I hope that you're friends with me!

- Internet (V=websites, E=a link from one site to another)
  Is this a directed graph? yes

I have a link to Rebecca Black's video on my website, but there's no link on the youtube page back to my website.

- Molecule Diagrams in Chemistry (V=atoms, E=bonds)
- Subway system (V=stops, E=link between consecutive stops)
  Is this a directed graph?



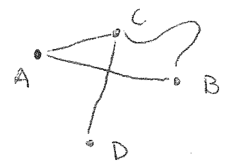
H_2O/water

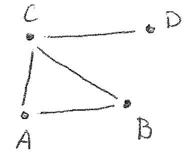
Graphs are used to model things in the real-world, especially when we have a collection of objects (eg people) and connections between some of them which are important (eg. friendships).

Untangling edges- planarity

A graph is a set of vertices with edges connecting them, so there are lots of different ways to draw the same graph.

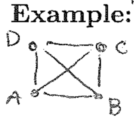
Example:



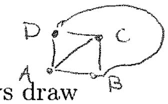


These graphs look pretty different, but they are the same! Can you say why? They have the same set of vertices and the edges connect the same vertices.

Example:



Can we redraw this so that the ~~e~~ edges don't intersect? Yep!



Question: Are all graphs like these two examples? Can we always draw them so that the edges don't intersect? Let's think of some graphs to test our guesses on.

Google 'planarity'. The point of the game is to untangle the edges of the given graph by moving vertices around.

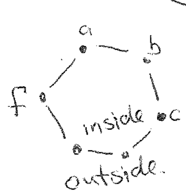
Look at the example sheet. Which of these graphs can be redrawn so that their edges aren't crossing? How are the bottom 4 graphs related?

Def: A graph is called planar if you can draw it so that no edges intersect. A graph is called non-planar if it's not possible to draw it in this way.

Proof that the utility graph is non-planar.

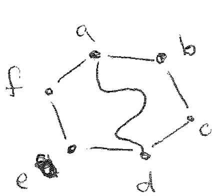
There are six vertices: {a, b, c, d, e, f}, and six of the edges are

a — b — c — d — e — f These form a loop. It's hard to show but true that

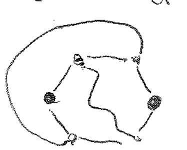


all loops divide the plane into an inside an outside.

All ~~the~~ edges have to be either inside or always outside, because otherwise they will cross another edge.



We can put one edge in on the inside, say between a and d, and this breaks up the inside into two pieces, and ~~so~~ for each edge we are missing, one vertex lies on one side of the edge between a and d, and the other vertex lies on the other side, so we can't connect them on the inside without crossing $\overline{ad}$. So the next two edges have to be on the outside... but we get trapped when we try to put in the last edge.



Again, the outside line divides the outside into 2 parts, and one vertex lies in each part. So the utility graph cannot be untangled!