SHORTEST PATHS IN GRAPHS March 2, 2013

Why would you want to find shortest paths?

- to one node on a graph – tourist searching for shortest route to his destination
- to all nodes on a graph – cable company that needs to wire all houses in a neighborhood using as little wiring as possible

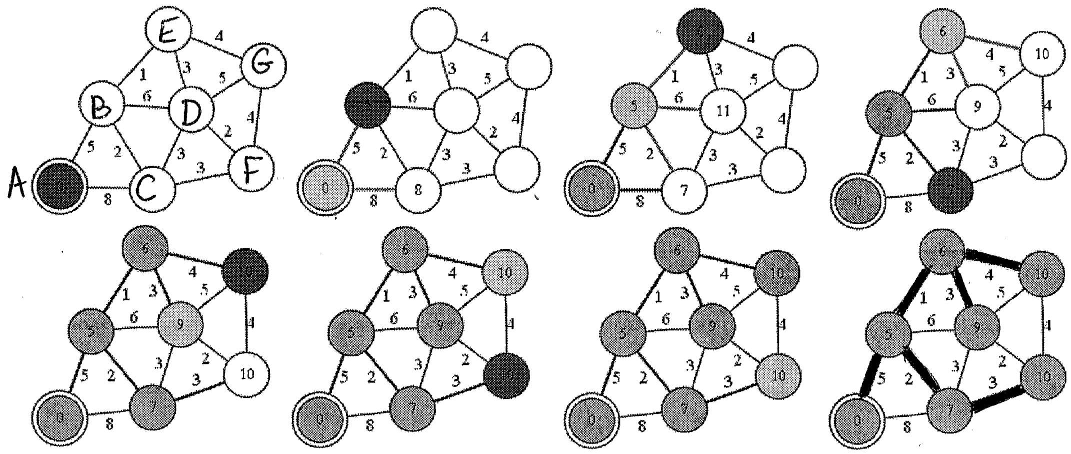
DIJKSTRA'S ALGORITHM

Facts: Runtime is $O(|V|^2)$. First introduced in 1957. Using min-priorty queue runs in $O(|E| + |V| \log |V|)$, found in 1984. Fastest known single-source shortest path algorithm.
Note: Assume weights are non-negative.
Idea: Assign initial distance value to each node and try to improve it step by step.

To find the shortest paths from an initial vertex to every other vertex of the graph, follow these steps:

- List all vertices as unvisited
- Set the distance to the initial node, say A, to 0 and the tentative distance to the rest of the nodes to ∞.
- Set A as current and visited (the rest of the nodes are still unvisited).
- For each neighboring (unvisited) node of A, check whether going through A improves the tentative distance to that node. If so, change the tentative distance value for that node.
- Select the unvisited node with smallest tentative distance and set it as new current node and repeat the process.
- The process stops when you have visited every node (or if you are looking for a shortest path to a specific node, the process stops when you visit that node).

Example:



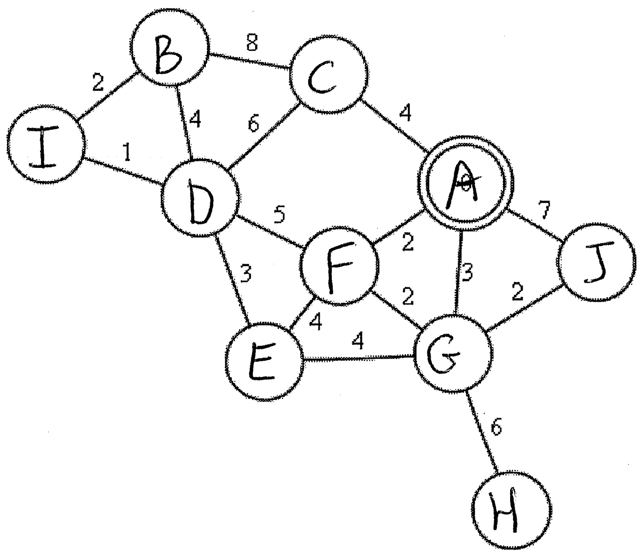
#	unvisited	visited	cur	A	B	C	D	E	F	G
0	{A,B,C,D,E,F,G}	{-}	–	(0,-)	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$
1	{B,C,D,E,F,G}	{A}	A		$(5, A)$	$(8, A)$	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$
2	{C,D,E,F,G}	{A,B}	B			$(7, B)$	$(11, B)$	$(6, B)$	$(\infty, -)$	$(\infty, -)$
3	{C,D,F,G}	{A,B,E}	E			$(7, B)$	$(9, E)$		$(\infty, -)$	$(10, E)$
4	{D,F,G}	{A,B,C,E}	C				$(9, E)$		$(10, C)$	$(10, E)$
5	{F,G}	{A,B,C,D,E}	D						$(10, C)$	$(10, E)$
6	{G}	{A,B,C,D,E,F}	F							$(10, E)$
7	{-}	{A,B,C,D,E,F,G}	G							

Question: What does it mean if we end up with one of the nodes having infinity for distance?
Answer: There is no path from the source node to this node.

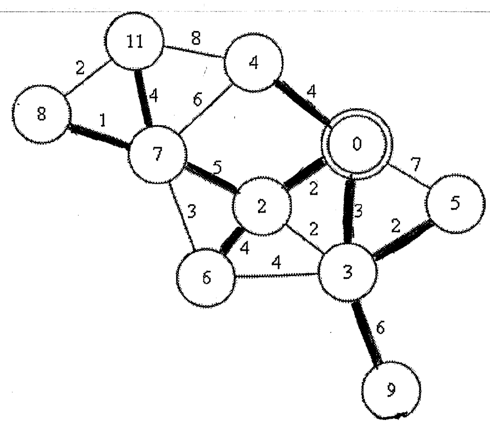
Question: What is special about the sub-graph we get?
Answer: It has no loops – it is a tree, called *minimal spanning tree.*

Activity 1: Watch `http://www.youtube.com/watch?v=dS1Di2ZHl4k&NR=1&feature=endscreen`

Activity 2: Find the minimal spanning tree of



Activity solution:



A (0,-)
F (2,A)
G (3,A)
C (4,A)
J (5,G)
E (6,F)
D (7,F)
I (8,D)
H (9,G)
B (11,D)