# 1 Lecture 1 - Problems Set

## 1.1 Symmetries of an equilateral triangle

**Question 1.1.** Suppose we compose two transformations, $T1$ first then $T2$, and suppose also that we get the trivial transformation. An example of this situation would the two translations mentioned earlier: translate "to the left" and then "back to the right". A natural question that arises is whether we still get the trivial transformation if we compose the transformations in reverse order? In this translation example, the answer is clearly yes. Is always the case when we compose two transformations?

**Question 1.2.** Make those two symmetries explicit by finding the rotation angles. Let us call these symmetries $R2$ and $R3$, where $R2$ is obtained by rotating by a smaller counter-clockwise angle.

**Question 1.3.** *Consider the following questions:*

(i) What is special about $R3$? Reprhase this using the trivial transformation language. Let us look at the first symmetry R1 we have found: we rotate the triangle around its center by 120° counter-clockwise. What happens when we compose $R1$ and $R1$, that is to say we do $R1$ twice in a row. Compare the composition symmetry with $R2$.

(ii) What happens now if we compose $R1$ three times in a row? Compare with $R3$.

(iii) What is the composition $R1$ then $R2$? How about $R2$ then $R1$? are they the same?

(iv) Rephrase the previous answer in terms of transformations that cancel each-other out and show how this gives another example of pairs of transformations that cancel each-other out no matter what order we compose them in.

**Question 1.4.** Suppose now we rotate the triangle 120° clockwise

(i) Show that this is also a symmetry of the triangle. Call it $R4$.

(ii) If we compose $R4$ with $R1$, we get the trivial transformation because that ends up rotating the triangle 120° counter-clockwise back to its original orientation. But we already know that composing $R1$ and $R2$ also give the trivial transformation. What kind of relationship is there between $R2$ and $R4$?

**Question 1.5.** Show that in that case that the reflection axis goes through A and the middle of segment [BC] – you can use the fact that the triangle is equilateral and hence all 3 corners make 60* angles.

**Question 1.6.** What is $X_A \circ X_B$?

**Question 1.7.** What about $X_C \circ X_A$ and $X_C \circ X_B$?

**Question 1.8.** What is $X_A \circ R2$? Then compare $X_A \circ R2$ with $R2 \circ X_A$.

**Question 1.9.** List all possible symmetries of a square.

**Question 1.10.** Can you do this for a regular $n$-gon for $n \geq 5$? Do we get more symmetries or less? Pick a particular $n$ and list all possible symmetries of the $n$-gon?

## 1.2 Permutations as a representation of symmetries

### 1.2.1 Permutations of 3 letters

**Question 1.11.** What are the permutation notations for $X_B$ and $R1$?

**Question 1.12.** Write down $X_C \circ X_A$ and check that $X_C \circ X_B = R2$ as well.

**Question 1.13.** But how many permutations of A, B and C are there all in all?

**Question 1.14.** Is the same true about the square?

### 1.2.2 Types of permutations

**Question 1.15.** What is an example of a transposition in our equilateral triangle example?

**Question 1.16.** What is an example of a 3-cycle in our equilateral triangle example?

### 1.2.3 Generators

**Question 1.17.** Show that the rotations correspond in that identification to the permutations

$$(1234), (1234)(1234) = (13)(24), (1234)(1234)(1234) = (1432);$$

the diagonal reflections to (24) and (13); and the other reflections to (12)(34) and (14)(23).

**Question 1.18.** Write (1234) as a composition of (not necessarily disjoint) transpositions?