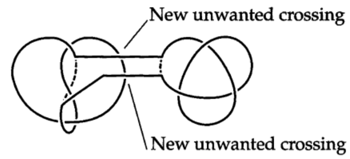
# Knot Theory Lesson 1 Notes

March 15th, 2014

**Section 1**
1. First definitions. Knot. Projection. Unknot. Trefoil. Figure-Eight. Take questions.
2. Explain what it means for two knots to be "the same".
3. General goals of knot theory: Find invariants. Find ways to simplify complicated projections.
4. Begin worksheet 1.

**Section 2**
1. Explain the connect sum of two knots. Choose arcs on the outside of the projections.



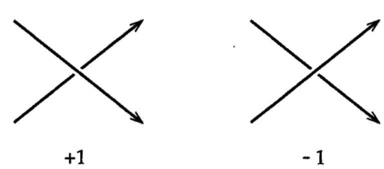

*Not* the composition of *J* and *K*.

2. Definitions: Composite knot. Factor Knot. Prime Knot.
3. What happens when we connect sum with the unknot?
4. Factorization is unique, and the unknot is not composite.
6. Begin worksheet 2.

**Section 3**
1. Define orientation, and explain the two possibilities under a connect sum.
2. Define invertible. Note that we don't have a good general method to determine invertibility.
3. Begin worksheet 3.

**Section 4**
1. Definitions: Ambient isotopy, planar isotopy.
2. Introduce the Reidemeister moves.
3. We claim Reidemeister + planar isotopies allow us to move between any two projections of the same knot.
4. Begin worksheet 4.

**Section 5**
1. Define invariant. Explain why we care.
2. Define linking number.




3. Explain how we could use Reidemeister to prove this is an invariant.
4. Begin worksheet 5.

**Section 6**
1. Explain how we haven't actually proven that every knot isn't the unknot.
2. Define strand. Introduce and define tricolorability: Each of the strands in the projections can be colored one of three different colors, so that at each crossing, either three different colors come together or all the same color comes together.
3. What is the problem with this definition?
4. Begin worksheet 6.
5. Introduce braids if this doesn't take us to the end of the period.

# Knot Theory Lesson 2 Notes

March 22nd, 2014

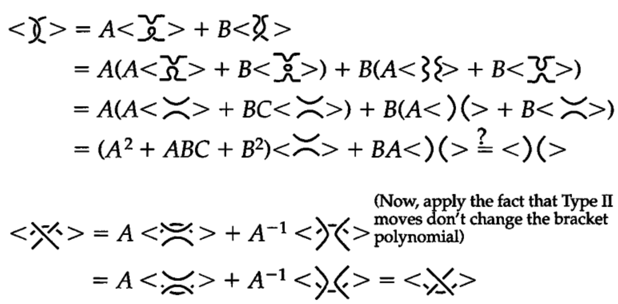
**Section 1**
1. Recap of last time: Definitions. What is a knot? What does it mean for two knots to be the same?
2. Review Worksheet. (Connect sum, Reidemeister, Invariants, Linking Number)

**Section 2**
1. Explain how we haven't actually proven that every knot isn't the unknot.
2. Define strand. Introduce and define tricolorability: Each of the strands in the projections can be colored one of three different colors, so that at each crossing, either three different colors come together or all the same color comes together.
3. What is the problem with this definition?
4. Begin worksheet 6.

**Section 3**
1. Unknotting number: We say that a knot has **unknotting number** $n$ if there exists a projection of the knot such that changing $n$ crossings makes it into the unknot, and there is no projection with fewer changes that would have changed it into the unknot.
2. Begin worksheet 7.
3. Why is unknotting number finite?
4. Worksheet 8.
5. Why isn't this a great invariant? In general it is incredibly hard to compute. Unsolved Questions. Simple proof that a knot with unknotting number 1 is prime. If a knot $K$ has $u(K) = 1$, is there always a crossing in any minimal crossing projection that we can change to make it the unknot?

**Section 4**
1. Seifert Surfaces: The goal is to find a surface in three dimensions which has boundary any given knot.
2. Worksheet 9.

**Section 5**
1. Polynomials as invariants. The Bracket Polynomial. Give out worksheet 10.
2. Calculation of Type II and II invariance:




3. The Jones Polynomial. Worksheet 11.

**Section 6**
1. Braids (if time)