

# Exercises

**Question 1:** Two dice are rolled, find the probability that the sum is

- a) equal to 1
- b) equal to 4
- c) less than 13

**Question 2:** A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

# Group Activities

1. Use 1 die and conduct a trial by rolling the die 100 times.  
What do you think is the probability of picking a 1?  
How many times did you roll or pick 1, 2, 3, 4, 5, 6?  
What is the percentage for each number?
  
2. Describe a situation where the probability is  $\frac{1}{2}$ .  
Describe a situation where the probability is  $\frac{1}{4}$ .  
Describe a situation where the probability is  $\frac{5}{9}$ .

# Group Activities

## 1. Geometric Distribution

The probability distribution of the number  $X$  of Bernoulli trials needed to get one success, supported on the set  $\{1, 2, 3, \dots\}$ . It's the probability that the first occurrence of success require  $k$  number of independent trials, each with success probability  $p$ . If the probability of success on each trial is  $p$ , then the probability that the  $k$ th trial (out of  $k$  trials) is the first success is:

$$\Pr(X = k) = (1 - p)^{k-1} p$$

for  $k = 1, 2, 3, \dots$

Find  $E(X)$

2. In probability theory, the coupon collector's problem describes the "collect all coupons and win" contests. It asks the following question: Suppose that there is an urn of  $n$  different coupons, from which coupons are being collected, equally likely, with replacement. What is the probability that more than  $t$  sample trials are needed to collect all  $n$  coupons?

An alternative statement is: Given  $n$  coupons, how many coupons do you expect you need to draw with replacement before having drawn each coupon at least once? The mathematical analysis of the problem reveals that the expected number of trials needed grows as  $n \cdot \log n$  as  $n$  grows. For example, when  $n = 50$  it takes about 225 trials to collect all 50 coupons.

Let  $T$  be the time to collect all  $n$  coupons, and let  $t_i$  be the time to collect the  $i$ -th coupon after  $i - 1$  coupons have been collected. Think of  $T$  and  $t_i$  as random variables. Find  $E(T)$  via calculating  $E(t_i)$  for each  $i$ .