Conditional Probability and Markov Chains

Conditional Probability

- Conditional Probability contains a condition that may limit the sample space for an event.
- You can write a conditional probability using the notation

$$P(B|A)$$

- This reads “the probability of event B, given event A”

Conditional Probability

The table shows the results of a class survey.
Find *P*(own a pet | female)

Do you own a pet?

	yes	no
female	8	6
male	5	7

14 females;
13 males

The condition female limits the sample space to 14 possible outcomes.

Of the 14 females, 8 own a pet.

Therefore, *P*(own a pet | female) equals $\frac{8}{14}$.

Conditional Probability

The table shows the results of a class survey.
Find *P*(wash the dishes | male)

Did you wash the dishes last night?

	yes	no
female	7	6
male	7	8

13 females;
15 males

The condition male limits the sample space to 15 possible outcomes.

Of the 15 males, 7 did the dishes.

Therefore, *P*(wash the dishes | male) $\frac{7}{15}$

Let's Try One

Using the data in the table, find the probability that a sample of not recycled waste was plastic. P(plastic | non-recycled)

The given condition limits the sample space to non-recycled waste.

A favorable outcome is non-recycled plastic.

Material	Recycled	Not Recycled
Paper	34.9	48.9
Metal	6.5	10.1
Glass	2.9	9.1
Plastic	1.1	20.4
Other	15.3	67.8

$$P(\text{plastic} \mid \text{non-recycled}) = \frac{20.4}{48.9 + 10.1 + 9.1 + 20.4 + 67.8}$$

$$= \frac{20.4}{156.3}$$

$$\approx 0.13$$

The probability that the non-recycled waste was plastic is about 13%.

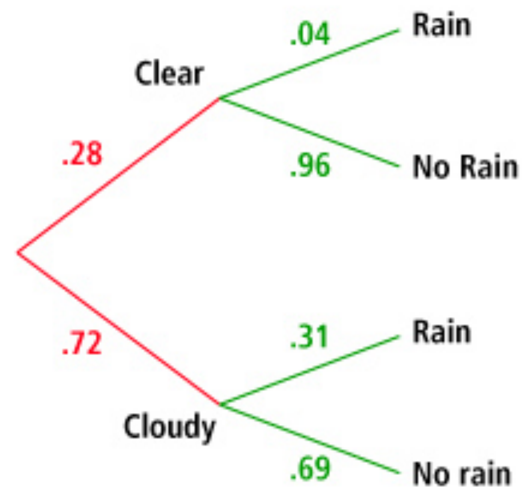
Conditional Probability Formula

- For any two events A and B from a sample space with P(A) does not equal zero

$$P(B|A) = \frac{P(AandB)}{P(A)}$$

Using Tree Diagrams

Jim created the tree diagram after examining years of weather observations in his hometown. The diagram shows the probability of whether a day will begin clear or cloudy, and then the probability of rain on days that begin clear and cloudy.



a. Find the probability that a day will start out clear, and then will rain.

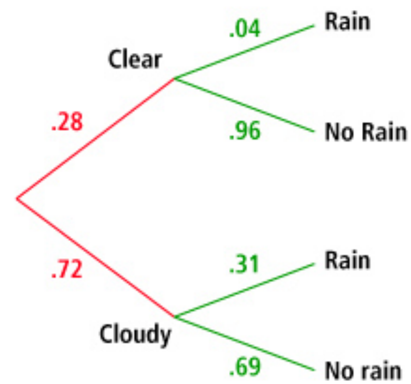
The path containing clear and rain represents days that start out clear and then will rain.

$$P(\text{clear and rain}) = P(\text{rain} \mid \text{clear}) \cdot P(\text{clear})$$
$$= 0.04 \cdot 0.28$$
$$= 0.011$$

The probability that a day will start out clear and then rain is about 1%.

Conditional Probability

(continued)



b. Find the probability that it will not rain on any given day.

The paths containing clear and no rain and cloudy and no rain both represent a day when it will not rain. Find the probability for both paths and add them.

P(clear and no rain) + *P*(cloudy and no rain) =
P(clear) • *P*(no rain | clear) + *P*(cloudy) • *P*(no rain | cloudy)
= 0.28(.96) + .72(.69)
= 0.7656

The probability that it will not rain on any given day is about 77%.

Let' s Try One

- A survey of Pleasanton Teenagers was given.
  - 60% of the responders have 1 sibling; 20% have 2 or more siblings
  - Of the responders with 0 siblings, 90% have their own room
  - Of the respondents with 1 sibling, 20% do not have their own room
  - Of the respondents with 2 siblings, 50% have their own room

Create a tree diagram and determine

A) *P*(own room | 0 siblings)

B) *P*(share room | 1 sibling)