Examples

1. In the preceding context, suppose the inspector is interested in the following events associated with the first five parts:
A = {The first two parts are good and the last three are bad.}
B = {Three good parts are produced before two bad ones are.}
These events cannot be described as above by the number of bad parts, but are describable by the sample space
S = {($x_1x_2x_3x_4x_5$): x_i = 0 or 1} where 1 means a good part and 0 means a bad part.
Find P (A) and P (B)

2. Consider the random experiment that describes the number of fire alarms that a certain station responds to in a week. Assume that it is known, based on historical data (via the frequency method discussed shortly), that the probabilities of exactly 0; 1; 2; 3; 4; 5 alarm responses are: 0.10; 0.15; 0.20; 0.25; 0.20; 0.10, respectively. (These numbers sum to 1.). The event A is the number of alarm responses is at least 3. Find P (A).

3. A flaw is detected in an underground cable of length 4000 feet. The flaw is equally likely to be located anywhere, which means that the probability of the flaw lying in any region A $\subseteq$ [0; 4000] is P (A) = (length of A)/4000. This rule obviously satisfies the definition of a probability. Find the probabilities of the following events:
B = the flaw is located within 200 feet of the center of the cable.
C = the flaw is located within 100 feet of either end of the cable.

4. A system fails when a defect occurs in one of 9 subsystems labeled 1, ..., 9 (only one defect occurs at a time). Let p_i denote the probability that the defect is in subsystem i. Suppose that each of the subsystems 4, 5, 6 is twice as likely to contain the defect as any one of the other subsystems. This information tells us that $p_i = 2p_j$ for $i \in \{4, 5, 6\}$ and $j \notin \{4, 5, 6\}$ (for instance $p_4 = 2p_9$). From these equalities, Find all p_i.

5. A certain product was found to have two types of minor defects. The probability that an item of the product has only a type 1 defect is 0.2, and the probability that it has only a type 2 defect is 0.3. Also, the probability that it has both defects is 0.1. Find the probabilities of the following events:
A = {An item has either a type 1 defect or a type 2 defect.}
B = {An item does not have either of the defects.}
C = {An item has defect 1, but not defect 2.}
D = {An item has exactly one of the two defects.}

6. (Ordered Sampling With Replacement) Suppose a random sample of four parts is chosen one at a time with replacement from the lot of 15. (For instance, the parts may be inspected and replaced at four stages in the production.) Find the probability of the events:
A = {The first 2 parts are type 5 parts.}
B = {The first and last parts are of type 1, 2 or 3.}

7. (Ordered Sampling Without Replacement) Suppose in the preceding setting that a random sample of four parts is chosen one at a time without replacement from the lot of 15 parts. (For instance, an inspector analyzes four parts at once, or a destructive test is done on four parts.) Find the probabilities of the events:
A = {All four parts are of type 1, 2, or 3.}
B = {The first and last parts are from 1 to 5.}

8. Unordered Sampling. In the preceding setting, suppose four parts are randomly chosen without replacement from the lot of 15. Find the probabilities of the events:
A = {All four parts are of type 1 or 2.}
B = {Two parts are for X and two parts are for Y} where X = {1, 2, 3, 4, 5} and Y = {6, 7, 8, 9, 10}.

9. Consider the number of hurricanes X that hit the east coast of the United States in one year. Suppose that its probability function is as follows:

x	0	1	2	3	4	5	6	7
p (x)	0.05	0.15	0.22	0.26	0.14	0.08	0.07	0.03

Find the expected number of hurricanes in a year.