Markov Chains

A discrete time process {Xn, n = 0, 1, 2, . . .} with discrete state space Xn ∈ {0, 1, 2, . . ., N} is a Markov chain if it has the Markov property:
$P(X_{n+1}=j \mid X_n=i, X_{n-1}=i_{n-1}, \ldots, X_0=i_0) = P(X_{n+1}=j \mid X_n=i)$

In other words, "the past is conditionally independent of the future given the present state of the process" or "given the present state, the past contains no additional information on the future evolution of the system."

The Markov property is common in probability models because, by assumption, one supposes that the important variables for the system being modeled are all included in the state space.

Define $P_{ij} = P(X_{n+1}=j \mid X_n=i)$ as the one-step transition probabilities and P_{ij}^{n} gives the n-step transition probabilities. Then we have $P_{ij}^{n+m} = \sum_{k=0}^{N} P_{ik}^{n} P_{kj}^{m}$.

<u>Examples</u>

1. We have two states to describe health status: Healthy (H) and Sick (S). Suppose the probability of being health next year in the condition of being healthy this year is 0.8, and the probability of being healthy next year in the condition of being sick this year is 0.7.
Suppose Bob is healthy this year, find the probability that he is still healthy 3 years later.

2. Weather forecast.