Introduction to Probability

Probabilities are associated with experiments where the outcome is not known in advance or cannot be predicted.

For example, if you toss a coin, will you obtain a head or tail? If you roll a die will obtain 1, 2, 3, 4, 5 or 6?

Probability measures and quantifies "how likely" an event, related to these types of experiment, will happen. The value of a probability is a number between 0 and 1 inclusive. An event that cannot occur has a probability (of happening) equal to 0 and the probability of an event that is certain to occur has a probability equal to 1.

In order to quantify probabilities, we need to define the **sample space** of an experiment and the **events** that may be associated with that experiment.

Sample Space and Events

The **sample space** is the set of all possible outcomes in an experiment.

Example 1: If a die is rolled, the sample space S is given by
S = {1,2,3,4,5,6}

Example 2: If two coins are tossed, the sample space S is given by S = {HH, HT, TH, TT}, where H = head and T = tail.

Example 3: If two dice are rolled, the sample space S is given by S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

We define an **event** as some specific outcome of an experiment. An event is a subset of the sample space.

Example 4: A die is rolled (see example 1 above for the sample space). Let us define event E as the set of possible outcomes where the number on the face of the die is even. Event E is given by E = {2,4,6}.

Example 5: Two coins are tossed (see example 2 above for the sample space). Let us define event E as the set of possible outcomes where the number of head obtained is equal to one. Event E is given by E = {(HT), (TH)}.

Example 6: Two dice are rolled (see example 3 above for the sample space). Let us define event E as the set of possible outcomes where the sum of the numbers on the faces of the two dice is equal to four. Event E is given by E = {(1,3), (2,2), (3,1)}.

How to Calculate Probabilities?

Classical Probability Formula: It is based on the fact that all outcomes are equally likely.

$$P(E) = \frac{\text{Total number of outcomes in E}}{\text{Total number of outcomes in the sample space}}$$

Example 7: A die is rolled. Find the probability of getting a 3. The event of interest is "getting a 3", so E = {3}. The sample space S is given by S = {1,2,3,4,5,6}.

The number of possible outcomes in E is 1 and the number of possible outcomes in S is 6. Hence the probability of getting a 3 is P ("3") = 1 / 6.

Example 8: A die is rolled. Find the probability of getting an even number. The event of interest is "getting an even number", so E = {2,4,6}, the even numbers on a die.

The sample space S is given by S = {1,2,3,4,5,6}. The number of possible outcomes in E is 3 and the number of possible outcomes in S is 6. Hence the probability of getting a even number is P ("even") = 3 / 6 = 1 / 2.