2.2 A Set of Set Exercises

Like numbers, *functions*, and most other mathematical objects, there are many ways to perform operations on sets. One can compare them (with *inclusion* denoted by $\subset$ and $\subseteq$), combine them (with *union* denoted by $\cup$), and find common elements (with *intersection* denoted by $\cap$). Here we will concern ourselves with the **set difference**, denoted by $A - B$, where we remove elements of the set B from the set A. Specifically,

$$A - B = \{\text{All elements of A that are not elements of B}\}.$$

14. Determine the set $\{\diamondsuit, \heartsuit, \clubsuit, \spadesuit\} - \{\clubsuit, \spadesuit\}$.

15. Determine the set {●, ◆, ⬬, ▮, ■, ▲} − {Four sided shapes}.

16. Write out ten elements in the set {a,b,c,...,x,y,z} that have been left out by the ellipsis.

17. A good name for the set {a,b,c,...,x,y,z} is EnglishAlphabet. Determine the set EnglishAlphabet− {a,e,i,o,u}? What is a good name for the set we have removed? For the resulting set that is left?

For finite sets the ***cardinality*** of the set is the number of elements in the set. For example, the cardinality of the set EnglishAlphabet is 26.

18. Determine the cardinality of each of the sets in Investigation **14**, Investigation **15**, and Investigation **17**.

Each time we use set difference, it gives rise to statement from arithmetic. For example,

$$\text{DaysoftheWeek} - \text{Weekend} = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}$$

gives rise to the arithmetical statement:

$$7 - 2 = 5.$$

19. Determine the arithmetical statement that corresponds to the set difference in Investigation **14**.

20. Determine the arithmetical statement that corresponds to the set difference in Investigation **15**.

21. Determine the arithmetical statement that corresponds to the set difference in Investigation **17**.

2.3 Infinite Arithmetic

Now let us move to the infinite. In particular, we are interested whether there is an arithmetic of the infinite that is consistent and makes sense.

22. Can you think of an appropriate value for $\infty - \infty$ is? Explain.

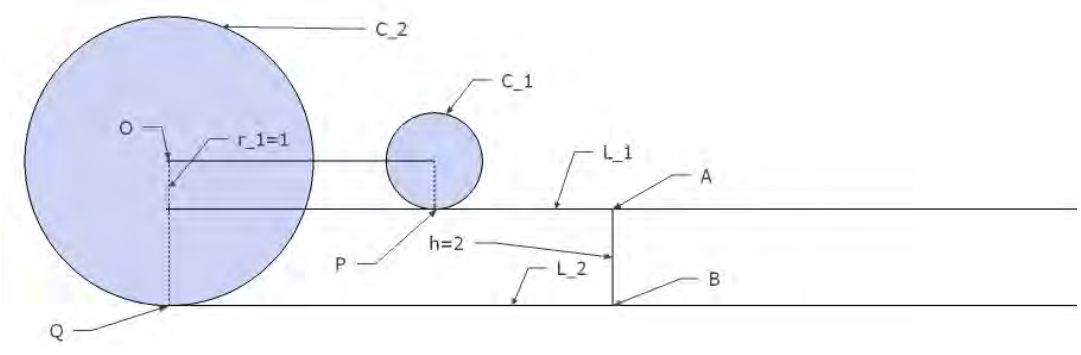

Figure 2.3: Two rolling wheels

23. For each of the sets $\{1, 2, 3, \ldots\}$ and $\{4, 5, 6, \ldots\}$ write out a dozen elements that have been left out by the ellipses.

24. From the infinite set $\{1, 2, 3, \ldots\}$ remove the infinite set $\{4, 5, 6, \ldots\}$. What is the remaining set and how large is it?

25. What does Investigation **24** suggest as a value for $\infty - \infty$?

26. Write out a dozen elements that have been left out of the set $\{5, 6, 7, \ldots\}$ by the ellipsis.

27. From the infinite set $\{1, 2, 3, \ldots\}$ remove the infinite set $\{5, 6, 7, \ldots\}$. What is the remaining set and how large is it?

28. What does Investigation **27** suggest as a value for $\infty - \infty$?

29. Can the investigations above be extended to suggest other values for $\infty - \infty$? Explain in detail what values are suggested and/or what the limitations are.

30. Write out a dozen elements that have been left out of the set $\{2, 4, 6, \ldots\}$ by the ellipsis.

31. From the infinite set $\{1, 2, 3, \ldots\}$ remove the infinite set $\{2, 4, 6, ...\}$. What is the remaining set and how large is it?

32. What does Investigation **31** suggest as a value for $\infty - \infty$?

33. What do the preceeding investigations suggest about our ability to discover a straightforward arithmetic for ∞?