Class 1, January 30: Large Numbers and Infinite Arithmetic

1. Ice Breaker: Came up with, shared and discussed the following: What's the weird number? funniest number? biggest number?

2. Large Numbers: Warm-up by asking "What's the biggest number you can think of?." Then have them compute in groups: a million seconds in a more reasonable unit of time (A: 11.6 days); a billion seconds in a more reasonable unit of time (A: 31.7 years); how many seconds they have been alive.

    > A parent asked their child "Whats the biggest number you can think of?" The child thinks about it and says "three-hundred and sixty seven." The parent asks "What about three-hundred and sixty eight?" to which the child responds "Oh. I knew I was close."

3. Infinite Arithmetic:

    (a) Write the following questions on the board. Have them discuss in groups and write their guesses.

    - Is ∞ even or odd?
    - What's $\infty - 1$?
    - What's $\infty - \infty$?
    - What's $0 \cdot \infty$?
    - What's $2 \cdot \infty$?
    - What's ∞/∞?

    (Source: https://www.artofmathematics.org/books/the-infinite)

    (b) Section 2.2 and 2.3 in the Art of Mathematics Infinity book: Beforehand, reminded them of how they learned to subtract in elementary school (seven apples, take away two apples, leaves you with five apples) and suggest that we could just apply this concept to infinity. Discuss the answers after they finish and go back to the questions on the board.

    (c) Discussion: Infinity is not a number like 2, 10^6, π or $3/4$. We need a different set of rules, of ideas about how the *concept* of infinity works. There are different "types" of objects in mathematics, each of which behaves in different ways, and it's a mistake to confuse ∞ with a number. When asked if $\infty \times 0 = 0$, most people say "Anything times zero is zero" but they are neglecting the implicit quantification – any *number* times zero is zero – and infinity is not a number.

4. Break!

5. Infinitesimals/Limits, infinity's crazy cousin: (I'm not sure how much of this makes sense, in terms of time or necessary mathematical background/maturity, but it's pretty flexible... Could learn about geometric series then compute Koch in beginning of next class...)

    (a) Present/discuss the following confusing "paradoxes":

    - $0 = 1$?

    $$\begin{aligned} 0 &= (1 + (-1)) + (1 + (-1)) + (1 + (-1)) + (1 + (-1)) + \cdots \\ &= 1 + ((-1) + 1) + ((-1) + 1) + ((-1) + 1) + ((-1) + 1) + \cdots \\ &= 1 \end{aligned}$$

    - $0.9999... = 1$?
    - Zeno's paradox and related puzzles

Class 2, February 6: Bijections, Cardinality, Continuum Hypothesis

1. Reintroductions: Icebreaker number 2?
2. Anything to finish from last time?
3. Cardinality worksheet (next page) and review
4. Bijections worksheet (next next page) and review. (Introduce with stuff about why this natural; figuring out if you have same number of stuff when you can't count)
   - There's a smaller infinity, all sets that have cardinality $\mathbb{N}$: $\mathbb{Q}, 2\mathbb{N}, 10^{\mathbb{N}}, \mathbb{N}^2$. Give proof that $|\mathbb{N}| = |\mathbb{Q}|$.
   - And there's a bigger infinity, all sets that have cardinality $\mathbb{R}$: any interval (give cool picture proof).
5. Give diagonalization argument: We want to prove that card($\mathbb{N}$) $\neq$card($\mathbb{R}$). Show that there is no bijection between the natural numbers ($\{1, 2, 3, ...\}$) and the real numbers (all decimal numbers). (Need to somehow preface with a discussion about proof by contradiction.)
6. State Continuum Hypothesis and independence result. (Need to preface with discussion about what independence means, which could actually be really fun.)