4. Putting the symmetries together

We would like to understand how the different symmetries of a given object relate to each other.

1. Pick one of the polygons to study. What happens if you rotate the polygon once to obtain a symmetry then rotate it again? What about if you rotate it once then reflect it once? Do you always get another symmetry? Why? We say that we have a *composition* of symmetries when we apply one symmetry then apply another (we have *composed* the symmetries).

2. If you reflect across a diagonal of the square, do you get a symmetry? Can you think of another way to obtain this symmetry with different transformations?

3. Try to find another example of this - what if you rotate 180 degrees? How else could you obtain this symmetry?

4. Are there any compositions of symmetries that end up doing nothing (namely, the object ends up back in its starting position)? Give some examples.

5. If you pick a particular symmetry can you always find another symmetry that composes with it to become the "do nothing" symmetry? Try to do this for every symmetry you thought of previously.

Extension Pick one of your polygons. Can you find a "minimal" (ie smallest) collection of symmetries so that every other symmetry of your polygon are just compositions of your minimal collection? What kind of relations do these symmetries satisfy? We call this minimal collection of symmetries the *generators* of the symmetries of your polygon.