CLOCKS

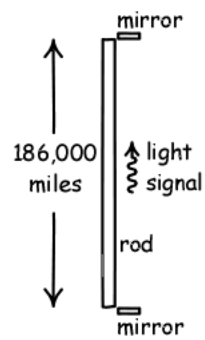
Math Explorer's Club

February 4, 2017

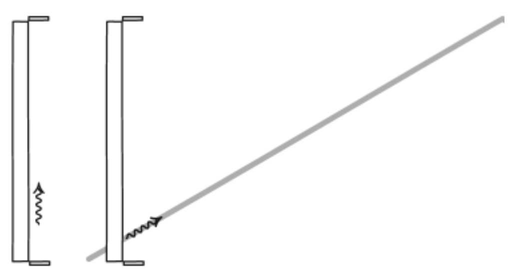
Spring 2017

Since the speed of light is the same for everyone, let's use the speed of light to build a super-accurate clock. In fact, this will be the simplest clock possible.

A light clock is an idealized clock that consists of a rod $186,000$ miles long with a mirror at each end. A light signal is reflected back and forth between the mirrors. Each arrival of the light signal at a mirror is a "tick" of the clock. Since light moves at $186,000$ miles per second, the light clock ticks once per second.



Now imagine that we take this clock and set it moving perpindicular to it's length. Now there's an added complication – when the light signal leaves one end of the rod headed towards the other end, the other end of the rod has moved! So the light signal has to chase the other end of the rod as it flees.



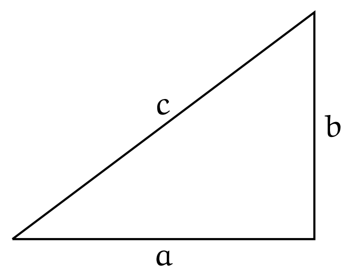
(Also see the animation here)

Why don't we notice the time dilation effect when we're driving? What about when we're flying on a plane? Let's put some numbers to this phenomenon.

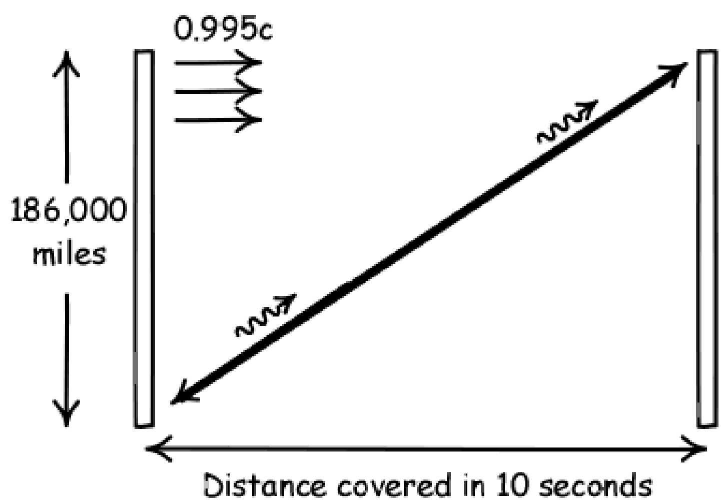
To do this activity, you will need only two very simple formulas.

$$\text{distance} = \text{speed} \cdot \text{time}$$

$$a^2 + b^2 = c^2$$



Imagine first that we put the light-clock in a spaceship that is flying at 99.5% of the speed of light away from you.



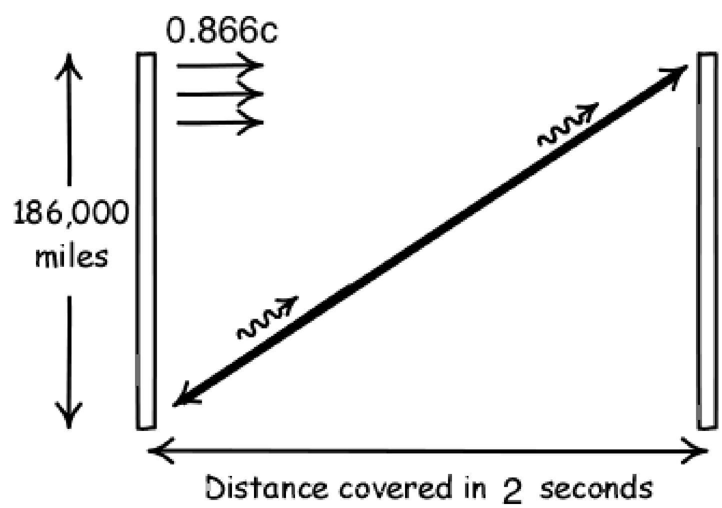
(1) If the clock is moving at 99.5% of the speed of light, how far does the clock move in 10 seconds?

(2) If the clock moves that distance in 10 seconds, how far does light have to travel between the two mirrors?

(3) How long does it take light to travel this distance?

(4) How much faster or slower does time pass for the moving clock, compared to one that is stationary?

Now what if the clock is in a spaceship that is flying at 86.6% the speed of light?



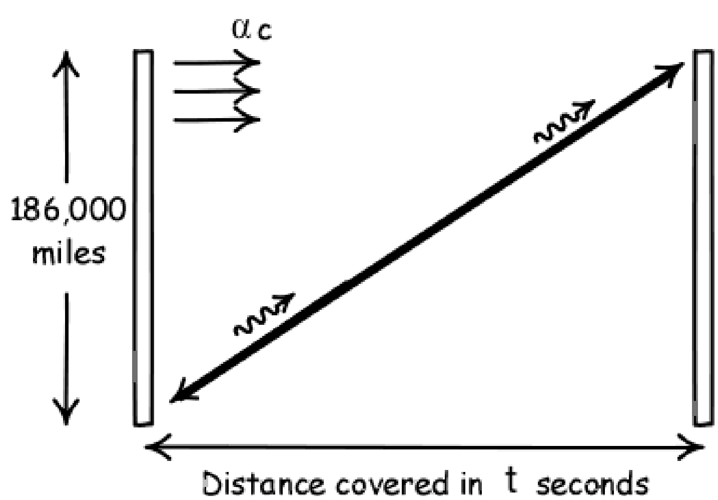
(5) If the clock is moving at 86.6% of the speed of light, how far does the clock move in 2 seconds?

(6) If the clock moves that distance in 2 seconds, how far does light have to travel between the two mirrors?

(7) How long does it take light to travel this distance?

(8) How much faster or slower does time pass for the moving clock, compared to one that is stationary?

So when the clock is moving *almost* at the speed of light – at 99.5% of the speed of light – time slows down significantly, but if you slow down just a little bit – to 86.6% of the speed of light, the time dilation isn't so dramatic. Let's see if we can figure out a formula for this.



Say that we're standing still, and the clock is moving away from us at a fraction α of the speed of light.

(9) If the clock is moving at a fraction α of the speed of light, how far does the clock move in t seconds?

(10) If the clock moves that distance in t seconds, how far does light have to travel between the two mirrors?

(11) Write a formula for the amount of time that it takes for light to travel this distance.

(12) This formula has t in it twice. Solve for t to figure out the time dilation factor.

This actually gives us a *formula* to determine the time dilation factor when the clock is moving at a fraction α of the speed of light.

So why don't you experience time dilation when you drive really fast in a car, or go on an airplane? Well, let's calculate it!

(13) Use your formula to fill in the table below.

Velocity	**Fraction α of Lightspeed**	**Time between ticks = $\frac{1}{\beta}$**
100mph*	.000000149	
575mph†	.000000814	
161,300 mi/sec	.866	2
185,400 mi/sec	.995	10

∗ A really fast car.
† An airplane.

(14) Based on this table, what do you think would happen if we travelled at the speed of light? How much time would pass between ticks on the clock, to an outside observer?