The Doppler Effect

Math Explorer's Club

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Spring 2017

Let's explore the Doppler effect!

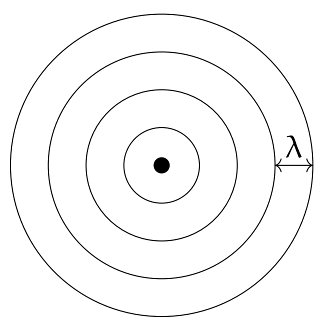
Like the speed of light, the speed of sound (in air) is a constant. The important thing here is that the sound wave moves with constant speed through the air, no matter if the source is moving or not, because the air is a stationary medium through which the sound waves move. Therefore, calculating the speed of sound in air is a good practice for relativistic doppler shifts.

Suppose that there is an ambulance moving away from you at speed $v_{\text{ambulence}}$, with a siren emitting sound with a frequency f and wavelength λ. We know that the frequency of sound that you hear will be different from the frequency of the sound that the ambulance produces, because you *hear* a different frequency. We can quantify that effect.

(1) Draw a picture to illustrate this scenario.

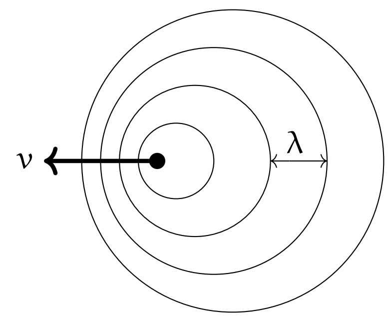
(2) What is the speed v_{sound} of the sound wave in terms of f and λ?

In the picture below, the concentric circles represent sound waves being emitted from the black dot in the center. When the dot isn't moving, the wavelengths arrive at regularly spaced intervals according to the frequency.



(4) How often are the sound waves emitted, if they have frequency f? This is called the **period** of the wave. Call it T.

Now, if the source is moving, it leaves the waves behind as it goes. The new picture looks something like this.



Let's figure out how the frequency you hear compares to the frequency that the siren emits. Let's call the frequency you hear f', and the corresponding wavelength λ'.

(5) Do you expect the sound to be higher frequency, or lower frequency? Why?

The wavelength is now longer than the old wavelength, because the ambulance is moving away. Let's find out how much longer.

(6) When one wave is emitted, we must wait one period (time T) before another one is emitted. How far does the ambulance travel in that time?

(7) The new wavelength λ' is the old wavelength plus the distance that the ambulance travelled. Write an equation for the new wavelength in terms of T, $v_{\text{ambulance}}$, and λ.

(8) In your answer to question (7), use your answer for question (3) to replace T by the variable f.

We know that the velocity of a wave is equal to the product of the frequency and wavelength.

$$v_{\text{sound}} = f' \cdot \lambda'$$

(8) Substitute your answer to question (8) into the equation above.

(9) Using the equation $v_{\text{sound}} = f \cdot \lambda$, substitute for λ in your answer.

(10) Multiply both sides of the equation by f and simplify.

(11) Solve for f'.

Now we have an equation for the relation of the two frequencies! Compare this to your answer in question (5). Is this what you expected? Why or why not?

(12) How do you think this situation would change if the ambulance was now moving *away* from you?

(13) How do you think this situation would change if the ambulance was moving faster than the speed of sound? (This is something that is forbidden with the speed of light, but not the speed of sound! Fighter jets regularly break the sound barrier.)