WORLDLINES

Math Explorer's Club

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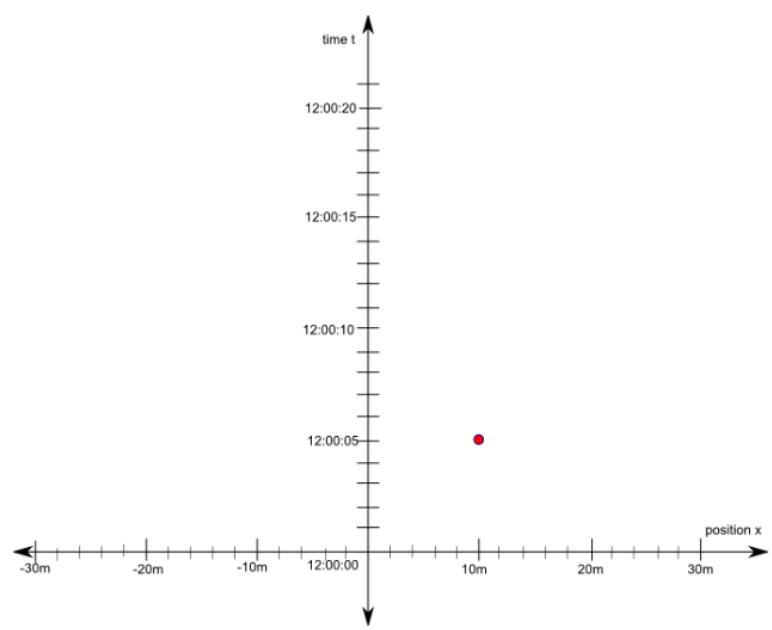
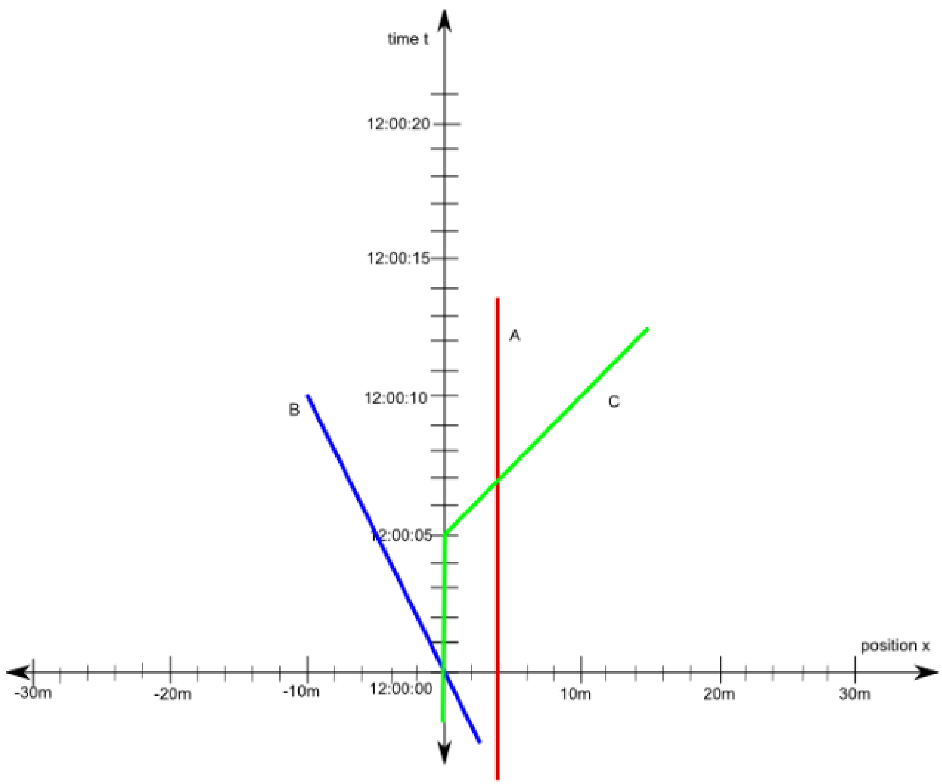
One of the basic philosophical insights in Einstein's theory of relativity is that the three dimensions of physical space and the one dimension of time should not be viewed as completely independent aspects of physical reality, but should instead be viewed as the two components of a unified four-dimensional spacetime on which all other physical objects reside.

Four-dimensional space is not that much more complicated to deal with than three-dimensional space; one simply has to replace three-dimensional vectors (x, y, z) by four-dimensional ones (x, y, z, t). We will focus on a simplified model of spacetime, in which we only consider one dimension of space, together with one dimension of time, leading to a spacetime that is only two dimensional. This allows for spacetime to be displayed on a page or screen in what is known as a **spacetime diagram**. It looks very similar to the familiar Cartesian plots used to graph functions, except that instead of having x and y axes, we have an x-axis (representing space) and a t-axis (representing time).



The points on a diagram of spacetime do not represent points in space, but rather **events** in spacetime. For example, on the diagram above, the dot represents an event 10 meters to the right of a fixed origin, at 5 seconds past noon.

A physical object (such as a human being) does not occupy just one point in spacetime; instead, it occupies a whole curve in spacetime, known as the **worldline** of that object.



For example, in the spacetime diagram above, the red line describes the worldlines of three people, A, B, and C. One of the advantages of spacetime diagrams is that it's easy to tell when or if two objects pass each other.

Consider the worldlines of the three people in the second spacetime diagram.

(1) Describe how Person A is moving through spacetime.

(2) Describe how Person B is moving through spacetime.

(3) Describe how Person C is moving through spacetime.

(4) How many seconds after noon will person C pass person A?

(5) How far from the origin will Person C and Person A meet?

(6) From Person B's perspective, he is stationary. Position the space coordinate relative to person B. Use a transparency to draw the worldline of Person A, *from the perspective of Person* B.

Alice is late for a flight, and she has to get from one end of a large airport to another. The airport is 500 meters long, and she walks at one meter per second. In the middle of the airport, there is a moving walkway that is 100 meters long, and moves in the direction Alice wants to go in at the speed of one meter per second; thus, if she is on the walkway while still walking, her net speed will in fact be two meters per second.

(6) Draw the axes for a spacetime diagram for the airport. Indicate where the walkway is.

(7) On this diagram, draw the worldline for Alice as she runs from one side of the airport to another.

Now suppose that Alice's shoelaces become untied in the middle of her dash from one side of the airport to the other. It takes her ten seconds to tie her shoes. She has three options for when to stop to tie her shoes: before she gets on the walkway, while she is on the walkway, and after she exits the walkway.

(8) Draw spacetime diagrams below to illustrate the three different choices Alice has (tying before, tying during, and tying after the walkway).

(9) What should Alice do? Which of the three choices will get her to the other side of the airport fastest?

(10) CHALLENGE: How does the answer to the previous question change if Alice can run twice as fast?