Topic 1 2.1

Question 1

mode MultipleSelection

text How can we approximate the slope of the tangent line to $f(x)$ at a point $x = a$? This is a Multiple selection question, so you need to check all of the answers that are correct.

correct-choice pick points close to $x = a$ and compute the slopes of the secant lines through $x = a$ and the chosen points

correct-choice pick 2 points on either side of $x = a$ and compute the average of the 2 corresponding secant line slopes

correct-choice plot $f(x)$ and draw an approximate tangent line at $x = a$ and use geometry to estimate its slope

comment The choices were:

- pick points close to $x = a$ and compute the slopes of the secant lines through $x = a$ and the chosen points
- pick 2 points on either side of $x = a$ and compute the average of the 2 corresponding secant line slopes
- plot $f(x)$ and draw an approximate tangent line at $x = a$ and use geometry to estimate its slope

All of the statements are correct. See each textbook example in section 2.1.

Question 2

mode Formula

text Let $s(t)$ be the distance travelled by a car at time t on the interval $[2, b]$. Give an expression of the average speed of the car from $t = 2$ to $t = b$.

answer `(s(b)-s(2))/(b-2)`

comment The average speed of the car is $\frac{s(b)-s(2)}{b-2}$.

Question 3

mode Formula

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answer `(s(b)-s(2))/(b-2)`

comment The average speed of the car is $\frac{s(b)-s(2)}{b-2}$.

Question 1

mode Multipart

text Let

$$f(x) = \begin{cases} x+4 & x < 1 \\ 2 & x = 1 \\ x^2 & x > 1 \end{cases}$$

Part (a)

mode numeric

text Compute $\lim_{x \to 1^-} f(x)$

answer 5

comment By graphing the function, we see that from the left, the limit is 5. Note that it does not matter that $f(1) = 2$ since we care about what happens *near* $x = 1$.

Part (b)

mode numeric

text Compute $\lim_{x \to 1^+} f(x)$

answer 1

comment By graphing the function, we see that from the right, the limit is 1. Note that it does not matter that $f(1) = 2$ since we care about what happens *near* $x = 1$.

Question 2

mode MultipleChoice

text If you are going to use a calculator to compute $\lim_{x \to a} f(x)$ for some function $f(x)$, the approach most likely to give the correct limit is to

choice compute $f(a)$

choice plug in values extremely close to $x = a$ on your calculator and look at what happens to $f(x)$ near $x = a$

correct-choice graph $f(x)$ on your calculator using different viewing rectangles to see what happens to $f(x)$ near $x = a$

choice plug in several values on your calculator near $x = a$ as long as $f(x)$ isn't periodic

comment The choices were:

- plug in values extremely close to $x = a$ on your calculator and look at what happens to $f(x)$ near $x = a$
- graph $f(x)$ on your calculator using different viewing rectangles to see what happens to $f(x)$ near $x = a$
- plug in several values on your calculator near $x = a$ as long as $f(x)$ isn't periodic

Using different viewing rectangles and plugging in values extremely close to $x = a$ is the best way to compute the limit on your calculator. See the textbook for examples on why the other methods don't always work.

Question 3

mode MultipleChoice

text If the limit $\lim_{x \to a} f(x)$ exists, then:

choice it equals $f(a)$

choice $f(x)$ must be defined at $x = a$

correct-choice it must be equal to the right hand limit as $x \to a$

choice $f(x)$ cannot continue to oscillate with a fixed amplitude or increase at a constant rate as $x \to a$

comment The choices were:

- it equals $f(a)$
- $f(x)$ must be defined at $x = a$
- it must be equal to the right hand limit as $x \to a$
- $f(x)$ cannot continue to oscillate with a fixed amplitude or increase at a constant rate as $x \to a$

Of the choices, this limit of $f(x)$ must be the same as the limit as $x \to a$ from the right. It also must be the same as the limit as $x \to a$ from the left. The important thing is to remember that is does not matter what happens at $f(a)$, but rather what happens to $f(x)$ *near* $x = a$.

Question 1

mode Numeric

text Compute $\lim_{x\to 5} x^2 - 9x + 2$.

answer `-18`

comment By the direct substitution property, the limit is -18.

Question 2

mode numeric

text Compute $\lim_{x\to 2} \frac{x^4-16}{x-2}$

answer `32`

comment We can factor the numerator and we have:

$$\begin{aligned} \lim_{x\to 2} \frac{x^4-16}{x-2} &= \lim_{x\to 2} \frac{(x^2-4)(x^2+4)}{x-2} \\ &= \lim_{x\to 2} \frac{(x-2)(x+2)(x^2+4)}{x-2} \\ &= \lim_{x\to 2} (x+2)(x^2+4) \\ &= 32 \qquad \text{(by the direct substitution property)} \end{aligned}$$

Question 3

mode TrueFalse

text If $\lim\limits_{x\to a} f(x) = 0$ and $\lim\limits_{x\to a} g(x) = 0$ then $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)}$ doesn't exist.

choice True

correct-choice False

comment False. Let $f(x) = x^4 - 16$ and $g(x) = x - 2$ as in the previous problem. Then as $x \to 2$, the limits of $f(x)$ and $g(x)$ go to zero, but we saw that using factoring techniques, the limit does exist.

Question 1

mode TrueFalse

text $f(x) = \frac{x^2-1}{x-1}$ is continuous on $[-2, 2]$.

choice True

correct-choice False

comment False. There is a discontinuity at $x = 1$.

Question 2

mode TrueFalse

text Let $P(t)$ be the cost of parking in Ithaca's parking garages for t hours. So $P(t) =$ \$0.50/hour or fraction thereof. $P(t)$ is continuous on $[1, 2]$.

choice True

correct-choice False

comment False. $P(1) = 0.50$ but $P(t) = 1.00$ for $1 < t \leq 2$. Therefore, $P(t)$ is not continuous on $[1, 2]$.

Question 3

mode MultipleSelection

text If $f(x)$ and $g(x)$ are continuous on $[a, b]$, which of the following are also continuous on [a,b]?

correct-choice $f(x) + g(x)$

correct-choice $f(x) \cdot g(x)$

choice $\frac{f(x)}{g(x)}$

correct-choice $f(x) + xg(x)$

comment The choices were:

- $f(x) + g(x)$
- $f(x) \cdot g(x)$
- $\frac{f(x)}{g(x)}$
- $f(x) + xg(x)$

All the choices are correct except for one. Note that $\frac{f(x)}{g(x)}$ is not continuous unless $g(x) \neq 0$ on $[a, b]$. This is a direct application of Theorem 4 in section 2.4.

Question 1

mode Multipart

text Compute the following limits.

Part (a)

mode numeric

text $\lim_{x \to \infty} \tan^{-1} x$

answer `3.14159265358979/2`

err `.01`

comment The answer is $\frac{\pi}{2}$ (see formula (6) in the text).

Part (b)

mode numeric

text $\lim_{x \to -\infty} \tan^{-1} x$

answer `-3.14159265358979/2`

err `.01`

comment The answer is $-\frac{\pi}{2}$ (see formula (6) in the text).

Question 2

mode TrueFalse

text When we write $\lim_{x \to a} f(x) = \infty$ this means that the limit exists and is a really big number.

choice True

correct-choice False

comment False. The limit does not exist. Writing the limit is equal to ∞ expresses the particular way in which the limit does not exist (see page 131 in the text).

Question 3

mode numeric

text Compute $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$

answer `1/2`

comment Solution is below:

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4} &= \lim_{x\to\infty} \frac{(x^3+5x)/x^3}{(2x^3-x^2+4)/x^3} \\
&= \lim_{x\to\infty} \frac{1+5/x^2}{2-1/x+4/x^3} \\
&= \frac{\lim_{x\to\infty}(1+5/x^2)}{\lim_{x\to\infty}(2-1/x+4/x^3)} \\
&= \frac{1}{2}
\end{aligned}$$

Question 4

mode Multipart

text For each of the following limits, choose if the limit implies that $f(x)$ has a vertical asymptote, horizontal asymptote, or neither.

Part (a)

mode Blanks

text $\lim_{x\to a} f(x) = -\infty$

```
vertical asymptote
horizontal asymptote
neither
```

Part (b)

mode Blanks

text $\lim_{x\to\infty} f(x) = a$

```
horizontal asymptote
vertical asymptote
neither
```

Part (c)

mode Blanks

text $\lim_{x\to-\infty} f(x) = a$

```
horizontal asymptote
vertical asymptote
neither
```

Part (d)

mode Blanks

text $\lim_{x\to a^+} f(x) = 5$

```
neither
horizontal asymptote
vertical asymptote
```

Part (e)

mode Blanks

text $\lim_{x \to a^-} f(x) = \infty$

```
vertical asymptote
horizontal asymptote
neither
```

Part (f)

mode Blanks

text $\lim_{x \to \infty} f(x) = -\infty$

```
neither
horizontal asymptote
vertical asymptote
```

Question 5

mode MultipleChoice

text Suppose $f(x)$ is a continuous function on $[1, 5]$. You know that $f(x) < 0$ on $[1, 2]$ and $f(x) > 0$ on $[4, 5]$ but do not know anything about $f(x)$ on the interval $(2, 4)$. Then for any function $f(x)$ that satisifes these conditions, $f(x)$ has a root in $[1, 5]$

correct-choice always

choice sometimes, but depends on $f(x)$

choice never

comment The choices were:

- always
- sometimes, but depends on $f(x)$
- never

Always. Since $f(x)$ is continuous, we know that it must pass through every value between $f(1)$ and $f(5)$ by the Intermediate Value Theorem. Therefore, at some point $x = a$, $f(a) = 0$ for $1 \leq a \leq 5$. Therefore, $f(x)$ has a root.

Question 1

mode numeric

text Compute the slope of the tangent line to $f(x) = x^2 + 5$ at the point $(2, 9)$.

answer 4

comment The slope of the tangent line is

$$\begin{aligned} m &= \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{x \to 2} \frac{x^2 + 5 - (2^2 + 5)}{x - 2} \\ &= \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \\ &= \lim_{x \to 2} x + 2 \\ &= 4 \end{aligned}$$

Question 2

mode MultipleChoice

text Let $s(t)$ be the distance traveled by a car at time t on the interval $[a, b]$. The average velocity of the car on $[a, b]$ is

correct-choice the slope of the secant line through $(a, f(a))$ and $(b, f(b))$

choice the slope of the tangent line at $x = a$

choice the slope of the tangent line at $x = b$

choice the slope of the tangent line at $x = \frac{a+b}{2}$

comment The choices were:

- the slope of the secant line through $(a, f(a))$ and $(b, f(b))$
- the slope of the tangent line at $x = a$
- the slope of the tangent line at $x = b$
- the slope of the tangent line at $x = \frac{a+b}{2}$

The average velocity is the slope of the secant line through $(a, f(a))$ and $(b, f(b))$ (see page 145 in the text).

Question 3

mode TrueFalse

text The average velocity over the time interval $[a, b]$ and the instantaneous velocity over the same time interval could be the same throughout the entire interval.

correct-choice True

choice False

comment True. Take any constant function. For example, let $f(x) = 5$ on $[1, 2]$. Then the average velocity is $\frac{5-5}{2-1} = 0$ and the instantaneous velocity at every point is also 0 since for any point a in $[1, 2]$,

$$\begin{aligned}\lim_{x \to a} \frac{f(x) - f(a)}{x - a} &= \lim_{x \to a} \frac{5 - 5}{x - a} \\ &= \lim_{x \to a} \frac{0}{x - a} = \lim_{x \to a} 0 \\ &= 0\end{aligned}$$

Question 4

mode Numeric

text The population of Ithaca increased from 21,887 in 1990 to 29,283 in 2000. Compute the average rate of change in the population from 1990 to 2000.

answer `(29283-21887)/10`

comment The average rate of change is $\frac{29283-21887}{2000-1990} = 739.6$.

Question 1

mode Multipart

text Let $f(x) = x^2$.

Part (a)

mode Blanks

text Set up the limit to compute the derivative of $f(x)$ at the point $(3, 9)$ by filling in the blank. $\lim_{h\to 0}($ `((3+h)^2-3^2)/h` $)$.

comment Using definition 2 in the text, the derivative is

$$\begin{aligned} f'(3) &= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} \\ &= \lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} \end{aligned}$$

Part (b)

mode numeric

text Compute the limit in (a).

answer 6

comment See the solution below.

$$\begin{aligned} \lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} &= \lim_{h\to 0} \frac{9 + 6h + h^2 - 9}{h} \\ &= \lim_{h\to 0} 6 + h \\ &= 6 \end{aligned}$$

Question 2

mode MultipleChoice

text The difference between the derivative of a function $f(x)$ at $x = a$ and the slope of the tangent line to $f(x)$ at $x = a$ is

choice the slope of the tangent line to $f(x)$ at $x = a$ is an approximate to the derivative of $f(x)$ at $x = a$

choice the derivative is a limit whereas the slope of the tangent line is not

choice the slope of the tangent line at $x = a$ is usually easier to compute than the derivative if the function is simple

correct-choice there is no difference

comment The choices were:

- the slope of the tangent line to $f(x)$ at $x = a$ is an approximate to the derivative of $f(x)$ at $x = a$
- the derivative is a limit whereas the slope of the tangent line is not
- the slope of the tangent line at $x = a$ is usually easier to compute than the derivative if the function is simple
- there is no difference

There is no difference between $f'(a)$ and the slope of the tangent line to $f(x)$ at $x = a$.

Question 3

mode Numeric

text The distance traveled in meters by a car at t seconds is give by $s(t) = 5t^2$. Compute the velocity of the car at $t = 2$ seconds.

answer 20

comment We need to compute the instantaneous velocity at $t = 2$:

$$\begin{aligned}
s'(2) &= \lim_{h\to 0} \frac{s(2+h) - s(2)}{h} \\
&= \lim_{h\to 0} \frac{5(2+h)^2 - 20}{h} \\
&= \lim_{h\to 0} \frac{20 + 20h + 5h^2 - 20}{h} \\
&= \lim_{h\to 0} 20 + 5h \\
&= 20
\end{aligned}$$

Question 1

mode Blanks

text The derivative of the function $f(x) = 7$ is $\boxed{\texttt{0}}$.

comment

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{7 - 7}{h} \\ &= \lim_{h \to 0} 0 \\ &= 0 \end{aligned}$$

Question 2

mode Blanks

text The derivative of the function $f(x) = 5x^2$ is $\boxed{\texttt{10*x}}$.

comment

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{5(x+h)^2 - 5x^2}{h} \\ &= \lim_{h \to 0} \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h} \\ &= \lim_{h \to 0} 10x + 5h \\ &= 10x \end{aligned}$$

Question 3

mode Multipart

text Let $f(x) = |x|$.

Part (a)

mode Blanks

text The derivative of $f(x)$ when $x < 0$ is $\boxed{\texttt{-1}}$.

comment Using the graph of $f(x) = |x|$ we know that the slope (derivative) to the left of zero is -1.

Part (b)

mode Blanks

text The derivative of $f(x)$ when $x > 0$ is $\boxed{\texttt{1}}$.

comment Using the graph of $f(x) = |x|$ we know that the slope (derivative) to the right of zero is 1.

Part (c)

mode Blanks

text What does (a) and (b) tell you about the derivative of $f(x)$ when $x = 0$?

```
Does not exist
is the same as (a)
is the same as (b)
is always constant
```
.

comment Does not exist. The derivative is a computation of a limit and therefore the right and left hand limits must be the same at $x = 0$. Part (a) tells us that the limit from the left is -1 and part (b) tells us that the limit from the right is 1. Therefore, the limit at $x = 0$ does not exist.

Question 4

mode Multipart

text Indicate which statements are true or false regarding a function $f(x)$.

Part (a)

mode TrueFalse

text If $f'(3)$ exists, then $f(x)$ is continuous at $x = 3$.

correct-choice True

choice False

comment True. Note that if $f'(3)$ exists, then $f(x)$ is continuous at $x = 3$ but the converse is not true (see page 163 in the text).

Part (b)

mode TrueFalse

text If $f(x)$ is continuous at $x = 3$ then $f'(3)$ exists.

choice True

correct-choice False

comment False. Note that if $f'(3)$ exists, then $f(x)$ is continuous at $x = 3$ but the converse is not true (see page 163 in the text).

Part (c)

mode TrueFalse

text If $f(x)$ has a vertical tangent at $x = 5$, then $f'(5)$ does not exist.

correct-choice True

choice False

comment True. A vertical tangent implies the derivative does not exist at that point.

Part (d)

mode TrueFalse

text If $f(x)$ has a cusp at $x = 5$, then $f'(5)$ does not exist.

correct-choice True

choice False

comment True. A cusp implies the derivative does not exist at that point.

Question 5

mode Multipart

text Indicate whether each of the following statements are true or false.

Part (a)

mode TrueFalse

text The second derivative, $f''(x)$, can be interpreted as the acceleration of an object where $f(x)$ measures the distance travelled of an object.

correct-choice True

choice False

comment True. See page 165 in the text.

Part (b)

mode TrueFalse

text $f''(x)$ is a rate of change of a rate of change

correct-choice True

choice False

comment True. See page 165 in the text.

Part (c)

mode TrueFalse

text $f''(x)$ is the derivative of $f'(x)$

correct-choice True

choice False

comment True. See page 165 in the text.

Question 1

mode Multipart

text Suppose you want to use a linear approximation to estimate the value of $(9.1)^{0.5}$. Answer the following questions.

Part (a)

mode Blanks

text The tangent line to $f(x) = x^{0.5}$ at $x = 9$ is a

```
valid linear approximation
invalid linear approximation
```

Part (b)

mode Blanks

text The tangent line to $f(x) = (9.1)^x$ at $x = .3$ is a

```
valid linear approximation
invalid linear approximation
```

Part (c)

mode MultipleChoice

text Which of the following approximations could be calculated without the use of a calculator?

correct-choice $f(x) = x^{0.5}$ at $x = 9$

choice $f(x) = (9.1)^x$ at $x = .3$

comment $f(x) = x^{0.5}$ at $x = 9$ can be calculated without a calculator.

Question 2

mode TrueFalse

text The linear approximation to a function $f(x)$ at a point $x = a$ is just the tangent line to $f(x)$ at $x = a$.

correct-choice True

choice False

comment True. The linear approximation at a point to a function *is* the tangent line at that point.

Question 3

mode TrueFalse

text Using the linear approximation to $f(x) = \cos x$ at the point $x = 1$ to estimate the value of $\cos(1.03)$ works well because $\cos x$ looks like a line when you zoom in on it.

correct-choice True

choice False

comment True. Linear approximation works because functions look like lines very close up.

Question 4

mode MultipleChoice

text If the tangent line to $f(x)$ at a point $x = 1$ is below $f(x)$, then using the linear approximation to estimate $f(1.1)$

choice could be an overestimate of the true value of $f(1.1)$

correct-choice could be an underestimate of the true value of $f(1.1)$

choice could be an overestimate or underestimate of $f(1.1)$ but depends on $f(x)$

comment The choices were:

- could be an overestimate of the true value of $f(1.1)$
- could be an underestimate of the true value of $f(1.1)$
- could be an overestimate or underestimate of $f(1.1)$ but depends on $f(x)$

Underestimate. This is because the tangent line is lower than the function $f(x)$.

Question 1

mode Blanks

text If $f(x) = x^3$ then $f'(x) =$ `3*x^2`.

comment Using the power rule, $f'(x) = 3x^2$.

Question 2

mode Blanks

text If $f(x) = e^x + 2$ then $f'(x) =$ `exp(x)`.

NOTE: If your answer involves e^x you should use “exp(x)” in your typed response.

comment Using the sum rule, $f'(x) = \frac{d}{dx}e^x + \frac{d}{dx}2 = e^x$.

Question 1

mode Formula

text Compute $\frac{d}{dx}(x^2e^x)$.

NOTE: If your answer involves e^x you should use "exp(x)" in your typed response.

answer `x^2*exp(x) + exp(x)*(2*x)`

comment Using the product rule,

$$\begin{aligned} f'(x) &= x^2(\frac{d}{dx}e^x) + e^x\frac{d}{dx}x^2 \\ &= x^2e^x + 2xe^x \end{aligned}$$

Question 2

mode Formula

text Compute $\frac{d}{dx}\left(\frac{e^x}{x^2}\right)$.

NOTE: If your answer involves e^x you should use "exp(x)" in your typed response.

answer `(exp(x)*(x^2)-2*x*exp(x))/x^4`

comment Using the quotient rule,

$$\begin{aligned} f'(x) &= \frac{x^2\frac{d}{dx}e^x - e^x\frac{d}{dx}x^2}{(x^2)^2} \\ &= \frac{x^2e^x - 2xe^x}{x^4} \end{aligned}$$

Question 1

mode Formula
text Compute $\frac{d}{dx}(2\sin x)$.
answer `2*cos(x)`

Question 2

mode Formula
text Compute $\frac{d}{dx}(3\sec x)$.
answer `3*sec(x)*tan(x)`

Question 3

mode numeric
text Compute the slope of the tangent line to $f(x) = \cos x$ at the point $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$.
answer `-sqrt(2)/2`
err `.01`
comment The slope of the tangent line is $f'\left(\frac{\pi}{4}\right)$. Since $f'(x) = -\sin x$, then $f'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

Question 4

mode Multipart
text Suppose you are asked to prove that $\frac{d}{dx}(\tan x) = \sec^2 x$. Indicate whether the following statements are true or false.

Part (a)

mode TrueFalse
text A possible way to prove that $\frac{d}{dx}(\tan x) = \sec^2 x$ is to start with the definition of the derivative and compute $\lim_{h\to 0} \frac{\tan(x+h)-\tan x}{h}$.
correct-choice True
choice False
comment True. The standard way the text uses to prove a formula of a derivative is to use the definition.

Part (b)

mode TrueFalse
text A possible way to prove that $\frac{d}{dx}(\tan x) = \sec^2 x$ is to use the fact that $\tan x = \frac{\sin x}{\cos x}$ and compute the derivative using the quotient rule.
correct-choice True
choice False
comment True. This method is perhaps easier than using the definition of the derivative and uses concepts we already know.

Part (c)

mode TrueFalse

text A possible way to prove that $\frac{d}{dx}(\tan x) = \sec^2 x$ is to use the fact the $\tan x = (\sin x) \cdot \left(\frac{1}{\cos x}\right)$ and compute the derivative using the product rule.

correct-choice True

choice False

comment True. This method is perhaps easiest and uses concepts we already know.

Topic 13 1.7

Question 1

mode MultipleChoice

text The parametric equations $x = 2\cos t$ and $y = t - \cos t$ for $0 \leq t \leq \pi$ give the following plot

http://mapleta.cit.cornell.edu:8080/classes/math111master/

Which statement below is true?

choice As t increases, the point (x, y) moves from left to right.

correct-choice As t increases, the point (x, y) moves from right to left.

comment The choices were:

- As t increases, the point (x, y) moves from left to right.
- As t increases, the point (x, y) moves from right to left.

The point (x, y) moves from right to left. For example, at $t = 0$, then $(x, y) = (2, -1)$ and at $t = \frac{\pi}{2}$, $(x, y) = (0, \frac{\pi}{2})$.

Question 2

mode MultipleChoice

text The difference in the motion described by between $x = \cos t$, $y = \sin t$ and $x = \cos 2t$, $y = \sin 2t$ is

choice nothing

correct-choice As t increases, the point (x, y) moves twice as fast for $x = \cos 2t$, $y = \sin 2t$

choice As t increases, the point (x, y) moves twice as fast for $x = \cos t$, $y = \sin t$

comment The choices were:

- nothing
- As t increases, the point (x, y) moves twice as fast for $x = \cos 2t$, $y = \sin 2t$
- As t increases, the point (x, y) moves twice as fast for $x = \cos t$, $y = \sin t$

As t increases, the point (x, y) moves twice as fast for $x = \cos 2t$, $y = \sin 2t$. See examples 2 and 3 in the text.

Question 1

mode Formula

text Compute $\frac{d}{dx}(\sqrt{x^2+e^x})$.

NOTE: If your answer involves e^x you should use "exp(x)" in your typed response.

answer `((1/2)/sqrt(x^2+exp(x)))*(2*x+exp(x))`

comment Using the chain rule with $f(x) = (x^2+e^x)^{1/2}$,

$$\begin{aligned} f'(x) &= \frac{1}{2}(x^2+e^x)^{(-1/2)} \cdot \frac{d}{dx}(x^2+e^x) \\ &= \frac{1}{2}(x^2+e^x)^{(-1/2)} \cdot (2x+e^x) \end{aligned}$$

Question 2

mode Formula

text Compute $\frac{d}{dx}(\sin(x^3))$.

answer `3*x^2*cos(x^3)`

comment Using the chain rule, $f'(x) = \cos(x^3) \cdot \frac{d}{dx}x^3 = 3x^2\cos(x^3)$.

Question 3

mode Numeric

text Compute the slope of the tangent line at the point $(0,1)$ of the parametric curve given by

$$x = \sin t \qquad y = \cos t$$

answer `0`

comment The slope of the tangent line is $\frac{dy}{dt}/\frac{dx}{dt} = -\frac{\sin t}{\cos t}$. The point $(0,1)$ corresponds to time $t=0$, so plugging in $t=0$ we have that the slope is $-\frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$.

Question 1

mode Multipart

text Complete the following parts.

Part (a)

mode Formula

text Use implicit differentiation to compute $\frac{dy}{dx}$ if $\sin y = x$.

answer `1/cos(y)`

comment The derivative is

$$\frac{dy}{dx}\cos y = 1$$

which implies that $\frac{dy}{dx} = \frac{1}{\cos y}$.

Part (b)

mode numeric

text Use your solution to the first question to find the slope of the tangent line to the curve $\sin y = x$ at the point $(0, \pi)$.

answer `-1`

err `.01`

comment Since $\frac{dy}{dx} = \frac{1}{\cos y}$, at the point $(0, \pi)$, the slope is $\frac{1}{\cos \pi} = -1$.

Question 1

mode Formula

text Compute $\frac{d}{dx}\ln(\sin x)$.

answer `cos(x)/sin(x)`

comment The derivative is $\frac{1}{\sin x}\cdot\frac{d}{dx}\sin x$ which gives $\frac{\cos x}{\sin x}$.

Question 2

mode TrueFalse

text $\dfrac{d}{dx}(x^e) = ex^{e-1}$.

correct-choice True

choice False

comment True. e is a constant, so applying the power rule, this statement is true.

Question 3

mode TrueFalse

text $\dfrac{d}{dx}(x^x) = x\cdot x^{x-1}$.

choice True

correct-choice False

comment False. The power rule does not apply since the exponent is not a constant. We need to use logarithmic differentiation on $y = x^x$ (start by taking ln of both sides):

$$\ln(y) = x\ln x$$

Then take derivatives on both sides with respect to x:

$$\begin{aligned}\frac{1}{y}\frac{dy}{dx} &= x\cdot\frac{1}{x} + \ln x\\ &= 1 + \ln x\end{aligned}$$

Solving for $\frac{dy}{dx}$ and substituting back for y we have:

$$\frac{dy}{dx} = x^x(1+\ln x)$$

Question 4

mode TrueFalse

text $\dfrac{d}{dx}(\ln|x|) = \dfrac{1}{|x|}$.

choice True

correct-choice False

comment False. This is not always true. Suppose $x < 0$. Then $\ln|x| = \ln(-x)$. Then

$$\frac{d}{dx}\ln(-x) = \frac{1}{-x}\cdot(-1)$$

which shows that $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$ for $x < 0$, not $\frac{1}{|x|}$.

Topic 17 3.8

Question 1

mode Multipart

text Complete the following sentence:

Part (a)

mode MultipleChoice

text The linear approximation or tangent line approximation of a function f at a point $x = a$ is the line through the point

correct-choice (a,f(a))

choice (a,f'(a))

Part (b)

mode Blanks

text ...with slope `f'(a)` / `f(a)` .

comment The linear approximation of a function f at a point $x = a$ is the line through $(a, f(a))$ with slope $f'(a)$.

Question 2

mode Blanks

text The equation of the linear approximation, $L(x)$, of f at a point $x = a$ is $L(x) =$ `f(a)+f'(a)(` / `f(a)+f'(x)(` / `ax+f'(a)`

comment $L(x) = f(a) + f'(a)(x - a)$.

Question 3

mode Matching

text Match the following quantities in the picture with the appropriate quantities below.

http://mapleta.cit.cornell.edu:8080/classes/math111master/

match a

with dx

match b

with Δy

match c

with dy

comment $a = dx = \Delta x$, $b = \Delta y$, $c = dy$ (see figure 5 on page 255 of the text).

Question 1

mode MultipleChoice
text When starting a related rates problem, which of the following would you do first?
correct-choice Find an equation that relates the quantities that are changing.
choice Compute the values of the quantities at a particular time.
choice Solve for all the unknowns.
comment The choices were:

- Find an equation that relates the quantities that are changing.
- Compute the values of the quantities at a particular time.
- Solve for all the unknowns.

The first thing to do when starting a related rates problem is to find an equation that relates the quantities that are changing. See page 267 of the text.

Question 2

mode MultipleChoice
text The length of a side, x, of a square of area, A, changes with time t. Then $\frac{dA}{dt} =$
correct-choice $2x\frac{dx}{dt}$
choice $2x$
choice $2t$
comment The choices were:

- $2x\frac{dx}{dt}$
- $2x$
- $2t$

Starting with the equation for the area, $A = x^2$, we take the derivative of both sides with respect to t and get $\frac{dA}{dt} = 2x\frac{dx}{dt}$.

Question 3

mode Multipart
text Recall the previous problem:
The length of a side, x, of a square of area, A, changes with time t. Find $\frac{dA}{dt}$.
Indicate whether we did or did not use the following tools for solving this related rates problem.

Part (a)

mode Blanks
text We [`did not use` / `did use`] the limit definition of the derivative to solve the previous related rates problem.

Part (b)

mode Blanks

text We [did use / did not use] the chain rule in solving the previous related rates problem.

Part (c)

mode Blanks

text We [did use / did not use] the power rule in solving the previous related rates problem.

Part (d)

mode Blanks

text We [did use / did not use] implicit differentiation in solving the previous related rates problem.

Part (e)

mode Blanks

text We [did not use / did use] derivatives of trigonometric functions in solving the previous related rates problem.

comment We used the chain rule, power rule, and implicit differentiation. We always need to use implicit differentiation with respect to a time variable t, and in doing so, we also use the chain rule. In this specific case, we also used the power rule to differentiate x^2.

Question 1

mode TrueFalse

text A continuous function on a closed interval must have an absolute maximum and an absolute minimum.

correct-choice True

choice False

comment True. This is the Extreme Value Theorem.

Question 2

mode TrueFalse

text A continuous function on an open interval must have an absolute maximum and an absolute minimum.

choice True

correct-choice False

comment False. $\frac{1}{x}$ is defined on the open interval $(0, \infty)$ and it has no extreme values. However, recall that a continuous function on a CLOSED interval must always have an absolute maximum or absolute minimum.

Question 3

mode TrueFalse

text If a function f has a local maximum or minimum at $x = c$ and if $f'(c)$ exists, then $f'(c) = 0$.

correct-choice True

choice False

comment True. This is Fermat's Theorem.

Question 4

mode Multipart

text Indicate whether the following statements are true or false.

Part (a)

mode TrueFalse

text When we look for extreme values of a function f, we should look at numbers c where $f'(c) = 0$.

correct-choice True

choice False

comment True. We need to look at critical numbers which include where $f'(c) = 0$.

Part (b)

mode TrueFalse

text When we look for extreme values of a function f, we should look at numbers c where $f'(c)$ does not exist.

correct-choice True

choice False

comment True. We need to look at critical numbers which include where $f'(c)$ does not exist.

Part (c)

mode TrueFalse

text When we look for extreme values of a function f, we should look at numbers c where c is an endpoint of the domain of f.

correct-choice True

choice False

comment True. We need to look at critical numbers as well as check the endpoints to see whether the absolute maximum or absolute minimum occurs there.

Part (d)

mode TrueFalse

text When we look for extreme values of a function f, we should look at numbers c where $f(c)$ does not exist.

choice True

correct-choice False

comment False. If $f(c)$ does not exist, then a maximum or minimum will not occur there, so we do not need to consider such points.

Question 1

mode TrueFalse

text The conclusion of the Mean Value Theorem says that the average rate of change equals the instantaneous rate of change at some point.

correct-choice True

choice False

comment True. The "some point" in the question is the number c in the statement of the Mean Value Theorem.

Question 2

mode numeric

text Using the conclusion of the Mean Value Theorem: $\frac{f(b)-f(a)}{b-a} = f'(c)$, what is $f(b) - f(a)$ if $f' = 0$ everywhere?

answer 0

comment Zero. If the derivative is always zero, then the function is a horizontal line and so $f(b) = f(a)$.

Question 3

mode Multipart

text Indicate whether the following statements are true or false.

Part (a)

mode TrueFalse

text When looking for local extrema, it is important to test all points where the derivative is zero.

correct-choice True

choice False

comment True. It is important to see where the derivative is zero, since at these points the function may change from increasing to decreasing.

Part (b)

mode TrueFalse

text When looking for local extrema, it is important to test all points where the derivative does not exist.

correct-choice True

choice False

comment True. It is important to check any points where the derivative does not exist, since at these points we may also find that the function changes from increasing to decreasing or vice versa.

Part (c)

mode TrueFalse

text When looking for local extrema, it is important to test all points where the second derivative is zero.

choice True

correct-choice False

comment False. The second derivative being zero may indicate an inflection point, but unless the first derivative is also zero at the same point we will not find a local extreme at place where the second derivative is zero.

Part (d)

mode TrueFalse

text When looking for local extrema, it is important to test any endpoint of the domain.

choice True

correct-choice False

comment False. Note that local extrema cannot occur at the endpoints (but absolute extrema can).

Question 4

mode MultipleChoice

text The reason we locate the points at which $f' = 0$ or f' does not exist to find local extrema is that

choice those must be the places where f has an absolute minimum or absolute maximum.

correct-choice everywhere else in the domain of f', either $f' > 0$ or $f' < 0$.

choice those are the roots of the function f

comment The choices were:

- those must be the places where f has an absolute minimum or absolute maximum.
- everywhere else in the domain of f', either $f' > 0$ or $f' < 0$.
- those are the roots of the function f

Everywhere else in the domain of f', either $f' > 0$ or $f' < 0$.

Question 5

mode TrueFalse

text It is possible to have a local minimum of f at $x = c$ if $f'(c) = 0$ and $f''(c) = 0$.

correct-choice True

choice False

comment True. See "note" on page 285 of the text. As an example to illustrate this point, let $f(x) = x^{2/3}(5 + x)$. Then

$$\begin{aligned} f'(x) &= \frac{5}{3}\left(\frac{2+x}{x^{1/3}}\right) \\ f''(x) &= \frac{10}{9}\left(\frac{x-1}{x^{4/3}}\right) \end{aligned}$$

Note that $f'(0) = 0$ and $f'(-2) = 0$ (the critical points). However, also note that $f''(0) = 0$. Using the first derivative test we see that $x = 0$ yields a local minimum even though $f'(0) = 0$ and $f''(0) = 0$.

Question 6

mode Multipart

text Indicate whether the following statements are true or false.

Part (a)

mode TrueFalse

text If $f'' > 0$ on an interval, then we know f is concave down on that interval.

choice True

correct-choice False

comment False. When $f'' > 0$ we know that f is concave up.

Part (b)

mode TrueFalse

text If $f'' > 0$ on an interval, then we know f' is increasing on that interval.

correct-choice True

choice False

comment True. When $f'' > 0$ then we know f' is increasing on that interval.

Part (c)

mode TrueFalse

text If $f'' > 0$ on an interval, then we know f has no local extrema on that interval.

choice True

correct-choice False

comment False. If f is concave up, then we could also have a local minimum or maximum of f on that interval ($f(x) = x^2$ for example is concave up and has a local minimum at $x = 0$).

Part (d)

mode TrueFalse

text If $f'' > 0$ on an interval, then we know that if f has a local extrema on this interval, then it must be a local minimum.

correct-choice True

choice False

comment True. If f is concave up and if also $f' = 0$ at some point in that interval (this is the second derivative test) then we know f has a local minimum in that interval.

Question 7

mode Multipart

text To find vertical asymptotes of a function f, it is important to check all the points where ...

Part (a)

mode TrueFalse

text ... $f' = 0$.

choice True

correct-choice False

comment False.

Part (b)

mode TrueFalse

text ... f' does not exist.

choice True

correct-choice False

comment False.

Part (c)

mode TrueFalse

text ... $f = 0$.

choice True

correct-choice False

comment False.

Part (d)

mode TrueFalse

text ... f is not defined.

correct-choice True

choice False

comment True.

Part (e)

mode TrueFalse

text ... f is not continuous.

correct-choice True

choice False

comment True.

Part (f)

mode TrueFalse

text ... f'' does not exist.

choice True

correct-choice False

comment False.

comment Recall that vertical asymptotes occur at places where f is not continuous or is not defined. You figure out if a function has vertical asymptotes by looking at the left and right limits of $f(x)$ as x approaches the point of discontinuity.

Question 1

mode TrueFalse

text To use L'Hopital's rule, one must be able to take the derivative of $\frac{f(x)}{g(x)}$, so it is important to be able to use the quotient rule.

choice True

correct-choice False

comment False. We take the derivative of the numerator and denominator separately when we use L'Hopital's rule.

Question 2

mode TrueFalse

text When using L'Hopital's rule to evaluate limits, if

$$\lim_{x \to a} \frac{f'(x)}{g'(x)}$$

does not exist, then

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

also does not exist.

choice True

correct-choice False

comment False. If the first limit does not exist, we obtain no information at all regarding the second limit. L'Hopital's rule states that IF $\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then both of the limits are equal. We cannot deduce anything about the relationships between the two limits otherwise.

Question 1

mode TrueFalse

text When you do an optimization problem, the critical point of the function will always tell you what the absolute maximum or absolute minimum is.

choice True

correct-choice False

comment False. First, the critical point of the function can be just a local minimum or local maximum. Second, you must check the endpoints to test for absolute extrema.

Question 2

mode Blanks

text Read the textbook examples 1-5 in section 4.6. For each example, indicate how the author justifies he has located the extreme value.

Example 1
```
Closed Interval Method
first derivative test
second derivative test
```

Example 2
```
first derivative test
Closed Interval Method
second derivative test
```

Example 3
```
first derivative test
Closed Interval Method
second derivative test
```

Example 4
```
Closed Interval Method
first derivative test
second derivative test
```

Example 5
```
Closed Interval Method
first derivative test
second derivative test
```

Topic 23 4.8

Question 1

mode TrueFalse

text The basic idea behind Newton's method can be summarized by the following quote:
"The tangent line is close to the curve, so the root of the tangent line is close to the root of the curve."

correct-choice True

choice False

comment True. This is the idea behind Newton's method. It's really just a linear approximation to the root.

Question 2

mode TrueFalse

text Newton's method for finding roots can be used to find the solution of

$$x^2 + 3x^5 - \sin x = 3\sqrt{x} + e^x$$

correct-choice True

choice False

comment True. We can use Newton's method to find the roots of

$$f(x) = x^2 + 3x^5 - \sin x - 3\sqrt{x} - e^x.$$

Question 1

mode TrueFalse

text The most general antiderivative of $\frac{1}{x}$ is $\ln(x) + C$.

choice True

correct-choice False

comment False. $\ln|x| + C$ also works.

Question 2

mode TrueFalse

text If $f'(x) = -\frac{1}{x^2}$ and $f(1) = 2$, then $f(x)$ must be $\frac{1}{x} + 1$ for all $x \neq 0$.

choice True

correct-choice False

comment False. We need to be careful when the domain is separated into different pieces ($x > 0$ and $x < 0$). For example, let

$$f(x) = \begin{cases} \frac{1}{x} + 1 & x > 0 \\ \frac{1}{x} + 3 & x < 0 \end{cases}$$

Then $f'(x) = -\frac{1}{x^2}$. Since the initial condition $f(1) = 2$ only refers to the part of the domain where $x > 0$, then $f(x) = \frac{1}{x} + 1$ only for $x > 0$. See the explanation of example 1b in the text.

Question 1

mode Multipart

text Suppose $f(x) \geq 0$ on $[2, 5]$. Indicate whether the following statements are true or false.

Part (a)

mode TrueFalse

text $(1)f(2) + (1)f(3) + (1)f(4)$ gives an estimate of the area under the graph of f.

correct-choice True

choice False

comment True.

Part (b)

mode TrueFalse

text $(1.5)f(2) + (1.5)f(5)$ gives an estimate of the area under the graph of f.

correct-choice True

choice False

comment True.

Part (c)

mode TrueFalse

text $(2)f(3) + (1)f(4)$ gives an estimate of the area under the graph of f.

correct-choice True

choice False

comment True.

Part (d)

mode TrueFalse

text $(3)f(3.5)$ gives an estimate of the area under the graph of f.

correct-choice True

choice False

comment True.

comment Note that all the options give an *estimate* of the area. Note that the subintervals do not all have to be the same size.

Question 2

mode MultipleChoice

text Suppose $v(t)$ is your velocity at time t and that $v(t) > 0$. Suppose you know $v(0), v(1), v(3), v(10)$. Then

$$(1)v(0) + (2)v(1) + (7)v(3)$$

represents an estimate of the

correct-choice total distance travelled from $t = 0$ to $t = 10$.

choice total distance travelled from $t = 0$ to $t = 3$.

choice the velocity from $t = 0$ to $t = 10$.

choice the velocity from $t = 0$ to $t = 3$.

comment The choices were:

- total distance travelled from $t = 0$ to $t = 10$.
- total distance travelled from $t = 0$ to $t = 3$.
- the velocity from $t = 0$ to $t = 10$.
- the velocity from $t = 0$ to $t = 3$.

This is an estimate of the area under $v(t)$ from $t = 0$ to $t = 10$ which is the total distance travelled over that time interval.

Question 1

mode TrueFalse

text If f is continuous on $[a,b]$, then $\int_a^b f(x)dx$ is the area bounded by the graph of f, the x-axis, and the lines $x = a, x = b$.

choice True

correct-choice False

comment False. The area is only defined for positive functions. If $f(x) < 0$, then the integral measures the *net* area.

Question 2

mode TrueFalse

text The area under the graph of $y = e^{-x^2}$ from $x = 0$ to $x = 1$ must be between $\frac{1}{e}$ and 1.

correct-choice True

choice False

comment True. See example 8 in the text.

Question 3

mode MultipleChoice

text Let $f(t)$ be the growth rate of a population at time t. Then $\int_{t_1}^{t_2} f(t)dt$ is

choice the change in growth rate between t_1 and t_2.

correct-choice the total population at t_2.

choice the change in the size of the population between t_1 and t_2.

comment The choices were:

- the change in growth rate between t_1 and t_2.
- the total population at t_2.
- the change in the size of the population between t_1 and t_2.

$\int_{t_1}^{t_2} f(t)dt$ is the the total population at t_2.

Question 1

mode TrueFalse

text Suppose a particle is moving in a straight line with position $s(t)$ and velocity $v(t)$ at time t for each t in $[t_1, t_2]$. Suppose that $s(t)$ is positive at time t. Then there must be some time t^* in $[t_1, t_2]$ so that

$$v(t^*)(t_2 - t_1) = s(t_2) - s(t_1)$$

correct-choice True

choice False

comment True. This is a consequence of the Mean Value Theorem and noting that $s'(t) = v(t)$.

Question 2

mode MultipleChoice

text If f is a continuous function, then $\int f(x)dx$

choice is a number.

choice precisely defines a function.

correct-choice is a family of functions.

comment The choices were:

- is a number.
- precisely defines a function.
- is a family of functions.

$\int f(x)dx$ is a family of functions. If $F(x)$ is an antiderivative of f, then $\int f(x)dx = F(x) + C$ defines a family of functions for different values of C.

Question 1

mode TrueFalse

text Consider $g(x) = \int_0^x \frac{1}{\sqrt{\theta^2+\theta}}$. If $x > 0$ then $g(x)$ is increasing.

correct-choice True

choice False

comment True. Using the Fundamental Theorem of Calculus, Part I, then

$$g'(x) = \frac{1}{\sqrt{x^2+x}}$$

which is always positive for $x > 0$. Since $g'(x) > 0$, then g is increasing.

Question 2

mode Multipart

text Let f be a continuous function on $[a, b]$ and $f(x) < 0$. Let $g(x) = \int_a^x f(t)dt$ for $a \leq x \leq b$. Then ...

Part (a)

mode TrueFalse

text ... $g(x)$ is defined for all x in $[a, b]$.

correct-choice True

choice False

comment True.

Part (b)

mode TrueFalse

text ... $g(x)$ is continuous on $[a, b]$.

correct-choice True

choice False

comment True.

Part (c)

mode TrueFalse

text ... $g(x)$ is a differentiable function on (a, b).

correct-choice True

choice False

comment True.

Part (d)

mode TrueFalse

text ... $g(x)$ is a decreasing function on $[a, b]$.

correct-choice True

choice False

comment True.

comment By the Fundamental Theorem of Calculus, Part I $g'(x) = f(x)$ and so that implies g is continuous and defined for all x in $[a, b]$. Since $f(x) < 0$, and $g'(x) = f(x)$, then $g'(x) < 0$ and so g is decreasing.

Question 3

mode MultipleChoice
text If $f'(x) = g(x)$ then $\int_a^x g(t)dt = f(x)$
choice always
correct-choice sometimes
choice never
comment The choices were:

- always
- sometimes
- never

Sometimes. By the Fundamental Theorem of Calculus, Part II if g is continuous then $\int_a^x g(t)dt = f(x) - f(a)$. Therefore, in order for $\int_a^x g(t)dt = f(x)$, then $g(x)$ must be continuous and $f(a) = 0$.

Question 1

mode MultipleChoice

text Suppose you use the substitution rule to evaluate the integral $\int \frac{\sin x}{(1+\cos x)^2} dx$ Then the BEST choice for u is

choice $u = \frac{\sin x}{(1+\cos x)^2}$

choice $u = \cos x$

choice $u = (1 + \cos x)^2$

correct-choice $u = 1 + \cos x$

comment The choices were:

- $u = \frac{\sin x}{(1+\cos x)^2}$
- $u = \cos x$
- $u = (1 + \cos x)^2$
- $u = 1 + \cos x$

$u = 1 + \cos x$. Although both $u = \cos x$ and $u = 1 + \cos x$ would work, the latter is the most efficient substitution since $u = 1 + \cos x$ gives us

$$-\int \frac{1}{u^2} du = \frac{1}{u} + C$$

and to finish, we substitute back in for u to get

$$\int \frac{\sin x}{(1 + \cos x)^2} dx = \frac{1}{1 + \cos x} + C$$

Question 2

mode MultipleChoice

text Substitution works because it "undoes" the

choice product rule

choice quotient rule

choice sum/difference rule

correct-choice chain rule

comment The choices were:

- product rule
- quotient rule
- sum/difference rule
- chain rule

Chain rule. The inside function of the chain rule is what becomes our "u" in the substitution.