HOW TO DEVELOP A TOP-TEN ALGORITHM IN THE 21ST CENTURY

Alex Townsend
Cornell University

(A talk for fun. Audience participation encouraged.)
DONGARRA’S TOP 10 LIST FROM 20TH CENTURY

1946: The Metropolis Algorithm
1947: Simplex Method
1950: Krylov Subspace Method
1951: The Decompositional Approach to Matrix Computations
1957: The Fortran Optimizing Compiler
1959: QR Algorithm
1962: Quicksort
1965: Fast Fourier Transform
1977: Integer Relation Detection
1987: Fast Multipole Method
HIGHAM’S TOP 10 LIST FROM 20TH CENTURY

1946: The Metropolis Algorithm
1947: Simplex Method
1950: Krylov Subspace Method
1951: The Decompositional Approach to Matrix Computations
1957: The Fortran Optimizing Compiler
1959: QR Algorithm
1962: Quicksort
1965: Fast Fourier Transform
1977: Integer Relation Detection
1987: Fast Multipole Method
1992: JPEG compression
1998: PageRank
1955: Kalman filter
1965: Quasi-Newton methods
A LIST OF TOP ALGORITHMIC DEVELOPMENTS 1900-2000

Timeline:

- Early electronic computers
- WW1
- WW2
- Colossus built
- Metropolis
- Simplex
- Householder
- Krylov
- QR algorithm
- Kalman filtering
- Fortran
- Quicksort
- FFT
- Quasi-Newton
- Integer relation
- IEEE arithmetic
- FMM
- JPEG
- PageRank

“Most influential numerical algorithms of 20th century”

(Complied list from Dongarra’s and Higham’s list + a few others)
HOW TO DEVELOP A TOP-TEN ALGORITHM IN THE 21ST CENTURY

Q1: What kind of person should I be?
   Highly collaborative or lone ranger. Young or old.

Q2: What kind of research environment should I be in?
   Academia or industry or government.

Q3: What kind of “products” should I create?
   Papers or conferences or industry/applications.
WHAT KIND OF PRODUCTS SHOULD I CREATE?

• Almost all the algorithms were published in traditional journal-style publications. (Exception: FORTRAN's programming manual.)

• Almost none of the algorithms were patented. (Exception: PageRank.)

• Not all were widely advertised by the "inventors". (Krylov & QR algorithm.)

• Some were instantly influential (e.g. FFT & FMM), others took decades (Metropolis-Hastings & Krylov).
## INFLUENTIAL PAPER: SHORT OR LONG

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Paper length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metropolis-Hastings algorithm</td>
<td>5</td>
</tr>
<tr>
<td>Simplex method</td>
<td>3</td>
</tr>
<tr>
<td>Krylov subspace method</td>
<td>28</td>
</tr>
<tr>
<td>Householder reflections</td>
<td>3</td>
</tr>
<tr>
<td>Kalman filter</td>
<td>10</td>
</tr>
<tr>
<td>Fortran compiler</td>
<td>37</td>
</tr>
<tr>
<td>QR algorithm</td>
<td>6</td>
</tr>
<tr>
<td>Quicksort</td>
<td>2</td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td>16</td>
</tr>
<tr>
<td>Fast Fourier transform</td>
<td>4</td>
</tr>
<tr>
<td>Integer relation detection</td>
<td>2</td>
</tr>
<tr>
<td>Fast multipole method</td>
<td>23</td>
</tr>
<tr>
<td>JPEG compression</td>
<td>17</td>
</tr>
<tr>
<td>PageRank</td>
<td>17</td>
</tr>
</tbody>
</table>

**Average: 12**  
**Median: 8**
LENGTH OF SIAM PUBLICATIONS

The mean number of pages > maximum allowed number of pages
INFLUENTIAL PAPER: INSTANT SUCCESS?

Fast Fourier transform:

James Cooley  John Tukey

Metropolis-Hastings algorithm:

Wilfred Hastings

Citation count of FFT 1965 paper

Citation count of Hastings 1970 paper
WHAT KIND OF RESEARCH ENVIRONMENT?

A selection of key individuals:

<table>
<thead>
<tr>
<th>Backus</th>
<th>Broyden</th>
<th>Greengard</th>
<th>von Neumann</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broyden</td>
<td>Cooley</td>
<td>Hastings</td>
<td>Page</td>
</tr>
<tr>
<td>Dantzig</td>
<td>Dantzig</td>
<td>Hestenes</td>
<td>Powell</td>
</tr>
<tr>
<td>Fletcher</td>
<td>Dantzig</td>
<td>Householder</td>
<td>Rokhlin</td>
</tr>
<tr>
<td>Francis</td>
<td>Dantzig</td>
<td>Kahan</td>
<td>Rutishauser</td>
</tr>
<tr>
<td>Givens</td>
<td>Dantzig</td>
<td>Tukey</td>
<td>Saad</td>
</tr>
<tr>
<td>Golub</td>
<td>Dantzig</td>
<td>Kantorovich</td>
<td>Wilkinson</td>
</tr>
<tr>
<td></td>
<td>Dantzig</td>
<td>Lanczos</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dantzig</td>
<td>Metropolis</td>
<td></td>
</tr>
</tbody>
</table>
RESEARCH ENVIRONMENT: INDUSTRY/GOVERNMENT

A selection of key individuals:

Backus
Broyden
Cooley
Dantzig
Fletcher
Francis
Givens
Golub

Greengard
Hastings
Hestenes
Householder
Kahan
Tukey
Kantorovich
Lanczos
Metropolis

von Neumann
Page
Powell
Rokhlin
Rutishauser
Saad
Wilkinson

Most had some involvement with industry and/or government.
RESEARCH ENVIRONMENT: ACADEMIA

A selection of key individuals:

Backus  
Broyden  
Cooley  
Dantzig  
Fletcher  
Francis  
Givens  
Golub  

Greengard  
Hastings  
Hestenes  
Householder  
Kahan  
Tukey  
Kantorovich  
Lanczos  
Metropolis  

von Neumann  
Page  
Powell  
Rokhlin  
Rutishauser  
Saad  
Wilkinson  

Most were academic professors.
RESEARCH ENVIRONMENT: MATH-RELATED PROFESSOR

A selection of key individuals:

Backus  Greengard  von Neumann
Broyden  Hastings  Page
Cooley  Hestenes  Powell
Dantzig  Householder  Rokhlin
Fletcher  Kahan  Rutishauser
Francis  Tukey  Saad
Givens  Kantorovich  Wilkinson
Golub  Lanczos  Metropolis

Most were professors in math, CS, and related fields.
RESEARCH ENVIRONMENT: ENGINEERS

A selection of key individuals:

Backus
Broyden
Cooley
Dantzig
Fletcher
Francis
Givens
Golub
Greengard
Hastings
Hestenes
Householder
Kahan
Tukey
Kantorovich
Lanczos
Metropolis
von Neumann
Page
Powell
Rokhlin
Rutishauser
Saad
Wilkinson

The others were mostly engineers.
A selection of key individuals:

<table>
<thead>
<tr>
<th>Backus</th>
<th>Greengard</th>
<th>von Neumann</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broyden</td>
<td>Hastings</td>
<td>Page</td>
</tr>
<tr>
<td>Cooley</td>
<td>Hestenes</td>
<td>Powell</td>
</tr>
<tr>
<td>Dantzig</td>
<td>Householder</td>
<td>Rokhlin</td>
</tr>
<tr>
<td>Fletcher</td>
<td>Kahan</td>
<td>Rutishauser</td>
</tr>
<tr>
<td>Francis</td>
<td>Tukey</td>
<td>Saad</td>
</tr>
<tr>
<td>Givens</td>
<td>Kantorovich</td>
<td>Wilkinson</td>
</tr>
<tr>
<td>Golub</td>
<td>Lanczos</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Metropolis</td>
<td></td>
</tr>
</tbody>
</table>

A good proportion worked in the USA.
WHAT KIND OF PERSON?

Lone ranger versus Collaborative

versus

Young versus Old
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metropolis algorithm</td>
<td>4</td>
</tr>
<tr>
<td>Simplex method</td>
<td>1</td>
</tr>
<tr>
<td>Krylov subspace method</td>
<td>2</td>
</tr>
<tr>
<td>Householder reflections</td>
<td>1</td>
</tr>
<tr>
<td>Kalman filter</td>
<td>1</td>
</tr>
<tr>
<td>Fortran compiler</td>
<td>0</td>
</tr>
<tr>
<td>QR algorithm</td>
<td>1</td>
</tr>
<tr>
<td>Quicksort</td>
<td>1</td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td>1</td>
</tr>
<tr>
<td>Fast Fourier transform</td>
<td>2</td>
</tr>
<tr>
<td>Integer relation detection</td>
<td>2</td>
</tr>
<tr>
<td>Fast multipole method</td>
<td>2</td>
</tr>
<tr>
<td>JPEG compression</td>
<td>1</td>
</tr>
<tr>
<td>PageRank</td>
<td>4</td>
</tr>
</tbody>
</table>

Average: 1.64
Median: 1
HOW OLD IS TOO OLD?

A selection of key individuals:

Backus, 30
Broyden, 32
Cooley, 39
Dantzig, 33
Fletcher, 24
Francis, 27
Givens, 47
Golub, 33
Greengard, 29
Hastings, 40
Hestenes, 46
Householder, 54
Kahan, 44
Tukey, 50
Kantorovich, 27
Lanczos, 59
Metropolis, 33
von Neumann, 44
Page, 28
Powell, 27
Rokhlin, 33
Rutishauser, 28
Saad, 36
Wilkinson, 46

Average: 37
Median: 33
In 1917, no-one knew the influential role of electronic computers... ... if an analogous event happens in the 2040s. We will all be too old.
SUMMARY

What kind of work?
• Write short papers (<10 pages).
• Publish in traditional journals.

What kind of research environment?
• Work closely with industry or government.
• In math-related academic field (favorably in the US).

What kind of person?
• Write papers alone or with one other.
• Better to be young, i.e., <45 years old
WHAT KIND OF WORK SHOULD I DO?

Simplified answer:

• Write short papers (<10 pages)

• Publish in traditional journals

• Many of the original papers seem to be motivated by applications. (Not always explicitly described.)
WHAT KIND OF RESEARCH ENVIRONMENT?

Simplified answer:

• Work closely with industry or government
• Become a professor in a math-related field
• It may help to work in the United States