MATH 2230, FALL 2014

HOMEWORK #1, DUE THURSDAY SEP 4 AT BEGINNING OF CLASS

Exercises from the book [Hubbard and Hubbard, 4th edition]:

- 0.2.1
- 0.3.1
- 1.1.3, 1.1.4, 1.1.9

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1. Let $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$, and define

$$\mathbb{R}\vec{v} := \left\{ \begin{bmatrix} ra \\ rb \end{bmatrix} \ : \ r \in \mathbb{R} \right\},$$

the set of scalar multiples of $\vec{v}$.

a. Show (which will always mean **prove**) that $\mathbb{R}\vec{v}$ is a subspace of $\mathbb{R}^2$. (Remember, you know *exactly* what's involved in this; if some vectors are not just in $\mathbb{R}^2$ but are in $\mathbb{R}\vec{v}$, and you combine them using our 2-,1-, or 0-ary operations, show that you end up inside V again.)

b. Say $\vec{v}' = \begin{bmatrix} a' \\ b' \end{bmatrix}$. When is $\mathbb{R}\vec{v} = \mathbb{R}\vec{v}'$? Don't just guess, prove your criterion. (When you want to check whether two sets A, B are equal, what you check is that $q \in A$ if and only if $q \in B$, or equivalently, $A \subseteq B$ and $B \subseteq A$.)

c. That says we're getting repeats: the same subspace is showing up for different $\vec{v}$. Give a "list" (in some sense – it's certainly infinite) of $\{\vec{v}_{\mathrm{good}}\}$ such that for any $\vec{v}$, there exists a *unique* $\vec{v}_{\mathrm{good}}$ in your list satisfying $\mathbb{R}\vec{v} = \mathbb{R}\vec{v}_{\mathrm{good}}$.

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2. Let X be a 4-element set, e.g. $\{1, 2, 19\frac{1}{2}, \text{Thanos}\}$.

a. If we don't put any operations on X at all, then every subset deserves to be called a subobject. How many subsets are there?

b. Now put one nullary operation α on X (like the $\vec{0}$ operation we have on $\mathbb{R}^n$). How many subobjects are there closed under it? Explain your answer(s).

c. Now put a second nullary operation, β, on X. How many subobjects are there closed under α *and* β? Explain your answer(s).