MATH 2230, FALL 2014

HOMEWORK #3, DUE THURSDAY SEP 18 AT BEGINNING OF CLASS

Exercises from the book [Hubbard and Hubbard, 4th edition]:

- 1.4.1, 1.4.7, 1.4.10c, 1.4.26
- 1.5.1, 1.5.3 (see below)

1.5.3a. Prove that any union of open sets is open.

Here's how to write such a thing:

> Let S be an "indexing" set, and for each $s \in S$, let U_s an open set in $\mathbb{R}^n$.
> Let $p \in \bigcup_{s \in S} U_s$ be a point in the union. We have to show that there exists an open ball of some positive radius $r > 0$ around p, completely contained in $\bigcup_{s \in S} U_s$.
> ...[use that the U_s are open]...

1.5.3b. Prove that any finite intersection of open sets is open.

Here's how to write such a thing:

> Let $U_1, \ldots, U_m$ be a list of open sets in $\mathbb{R}^n$.
> Let $p \in \bigcap_{i=1}^{n} U_i$ be a point in the intersection. We have to show that there exists an open ball of some positive radius $r > 0$ around p, completely contained in $\bigcup_{s \in S} U_s$.
> ...[use that the U_i are open]...

1.5.3c. Find a set $\{U_s : s \in S\}$ of open sets such that $\bigcap_{s \in S} U_s$ is (provably!) not open.

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1. Define a set $U \subseteq \mathbb{R}^n$ to be **copen** if for every $p \in U$, there exists a *closed* ball B of positive radius such that $p \in B \subseteq U$.

a. Prove that a copen set is open. (I.e. show that given the closed balls, we can find the required open balls.)

b. Prove that a open set is copen. (I.e. show that given the open balls, we can find the required closed balls.)