HW # 3 DUE THURSDAY, SEPTEMBER 21

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Recall for $X \subseteq \mathbb{C}^n$, blown up along $X \cap \mathbb{C}^{n-j}$, the blowup $\widetilde{X}$ lives in $\mathbb{C}^k \times \widetilde{\mathbb{C}}^j$, which naturally maps back down to X. Define the **exceptional divisor** as the preimage in $\widetilde{X}$ of the $\mathbb{C}^{n-j}$ we blew up. (We'll define "divisors" later.)

#1. Let $X = V(\langle x(y-1), y(y-1)\rangle)$. Compute the proper transform of X, when blowing up the plane at the origin. What are the prime components of X and $\widetilde{X}$?

#2. Let $X = V(\langle x_1x_3 - x_2^2\rangle)$. Compute the proper transform of X, when blowing up 3-space along the x_3-axis. What are the prime components of X and $\widetilde{X}$?

#3. Try it out in Macaulay 2. In particular, convince yourself that the following code is doing the job.

```
n = 3; j = 2; -- blow up n-space along x_1 = ...  = x_j = 0
R = QQ[x_1..x_n, y_1..y_j];
Bl = ideal exteriorPower(2, matrix {apply(j,i->x_(i+1)),apply(j,i->y_(i+1))});
irr = ideal apply(j,i->x_(i+1)); -- irrelevant ideal
IX = ideal {x_1*x_3 - x_2^2}; -- ideal of X
BX = saturate(I + Bl, irr); -- ideal of the blowup
E = irr + BX; -- ideal of the exceptional divisor
decompose E
```

With this doing the algebraic work for you, what are the prime components of the exceptional divisor? Describe them geometrically.

#4. If $S \geq J$ are a ring and ideal, define the **blowup algebra** $B(S,J) := \bigoplus_{n\in\mathbb{N}} t^n J^n \leq S[t^{(1)}]$, where S is put in degree 0 and t in degree 1. Compute $\mathtt{Projm}\ B(S,J)$ when $J = 0$.

#5. Compute $\mathtt{Projm}\ B(S,J)$ when $S = \mathbb{C}[x,y]$ and $J = \langle x, y\rangle$.

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