HW # 4 DUE THURSDAY, SEPTEMBER 28

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Recall that the **Rees algebra** of $S \geq J$ is

$$\mathrm{Rees} := \oplus_{n\in\mathbb{Z}} \begin{cases} St^n & \text{if } n \leq 0 \\ J^n t^n & \text{if } n \leq 0 \end{cases}$$

and the **normal cone** is $\mathrm{Rees}/\langle t^{-1}\rangle$.

Call a point $x \in \mathrm{Specm}\ S$ **regular** if its normal cone is just a polynomial ring, without any relations. Also, $\mathrm{Specm}\ S$ is **regular** if it is regular at every point.

#1. Let S be a quotient of a polynomial ring in finitely many variables (over a field). Show that Rees is also such.

#2. Use the `map, ker,` and `flatten entries gens` commands in Macaulay2 to give a presentation of a Rees algebra as a polynomial ring mod an ideal.

#3. Let $S = \mathbb{C}[x, y]/\langle xy\rangle$. For each point in the axes $\mathrm{Specm}\ S$, determine the normal cone. Which points are regular?

#4. Let $S = \mathbb{C}[x, y]/\langle x^2 - y^3\rangle$. For each point in the "cuspidal curve" $\mathrm{Specm}\ S$, determine the normal cone. Which points are regular?

#5. Draw the solution set of $y^2 = x^2(x + 1)$. Let $S = \mathbb{C}[x, y]/\langle y^2 - x^2(x + 1)\rangle$ be the corresponding ring. Show that S is a domain, but $\mathrm{gr}_{(0,0)}S$ is not.

#6. Continuing #5, show that S is not regular, but its blowup at $(0, 0)$ is.

Date: September 22, 2017.