MATH 6320 FINAL, SPRING 2017

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1. Let G be a finite group. Make the set $G^* := \mathrm{Hom}(G, \mathbb{C}^\times)$ into a group by multiplication of values.

a) Explain how to determine the structure of G^* (up to isomorphism) from the character table of G.

b) Prove that $G^{**} \cong G^*$. (Yes, that's right.)

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2. Compute the character table of the group G of signed 3×3 permutation matrices, i.e. of size $3!2^3$.

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3. Let $R = \mathbb{C}[[z]]$ be the ring of power series, considered as a $\mathbb{Z}$-graded ring (where $\deg z = 1$). Let $\mathcal{C}$ be the category of $\mathbb{Z}$-graded R-modules, i.e. where module homomorphisms preserve the grading.

a) Classify finitely generated graded R-modules.

b) Compute the groups $\mathrm{Ext}^\bullet(M, N)$ where M, N are two such modules.

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4. Let **Ab** be the category of finitely generated abelian groups, and $T := \mathrm{Hom}(\bullet, \mathbb{Z})$ the contravariant functor $\mathbf{Ab} \to \mathbf{Ab}$.

a) Show that T does not square to the identity (as it would if we replaced $\mathbb{Z}$ by $\mathbb{C}$).

b) Let $\mathcal{D}\mathbf{Ab}$ be the derived category, i.e. bounded chain complexes considered up to chain maps that induce isomorphisms on homology. To the best of your ability, justify the statement "$R^\bullet T : \mathcal{D}\mathbf{Ab} \to \mathcal{D}\mathbf{Ab}$ does square to the identity."

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5. Let $\mathbb{F}$ be a finite field of characteristic p, and $f \in \mathbb{F}$. Show that $x^p - f \in \mathbb{F}[x]$ factors completely.

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6. Let $\mathbb{F}$ be of characteristic p, and assume that $P : \mathbb{F} \to \mathbb{F}$, $x \mapsto x^p$ is not bijective. Show that the sequence $P^k(\mathbb{F})$ strictly decreases in k, forever.

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7. For $a, b \in \mathbb{N}$ show $\mathbb{Q}[\sqrt{a} + \sqrt{b}] = \mathbb{Q}[\sqrt{a}, \sqrt{b}]$.

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8. Compute the Galois group of the splitting field of $x^4 - 4$ over $\mathbb{Q}$. Find all the intermediate subfields.