1. Let $G$ be a finite group. Make the set $G^* := \text{Hom}(G, \mathbb{C}^*)$ into a group by multiplication of values.
   a) Explain how to determine the structure of $G^*$ (up to isomorphism) from the character table of $G$.
   b) Prove that $G^{**} \cong G^*$. (Yes, that’s right.)

2. Compute the character table of the group $G$ of signed $3 \times 3$ permutation matrices, i.e. of size $3!2^3$.

3. Let $R = \mathbb{C}[[z]]$ be the ring of power series, considered as a $\mathbb{Z}$-graded ring (where $\deg z = 1$). Let $\mathcal{C}$ be the category of $\mathbb{Z}$-graded $R$-modules, i.e. where module homomorphisms preserve the grading.
   a) Classify finitely generated graded $R$-modules.
   b) Compute the groups $\text{Ext}^*(M, N)$ where $M, N$ are two such modules.

4. Let $\text{Ab}$ be the category of finitely generated abelian groups, and $T := \text{Hom}(\bullet, \mathbb{Z})$ the contravariant functor $\text{Ab} \to \text{Ab}$.
   a) Show that $T$ does not square to the identity (as it would if we replaced $\mathbb{Z}$ by $\mathbb{C}$).
   b) Let $\mathcal{D}\text{Ab}$ be the derived category, i.e. bounded chain complexes considered up to chain maps that induce isomorphisms on homology. To the best of your ability, justify the statement “$R^*T : \mathcal{D}\text{Ab} \to \mathcal{D}\text{Ab}$ does square to the identity.”

5. Let $\mathbb{F}$ be a finite field of characteristic $p$, and $f \in \mathbb{F}$. Show that $x^p - f \in \mathbb{F}[x]$ factors completely.

6. Let $\mathbb{F}$ be of characteristic $p$, and assume that $P : \mathbb{F} \to \mathbb{F}, x \mapsto x^p$ is not bijective. Show that the sequence $P^k(\mathbb{F})$ strictly decreases in $k$, forever.

7. For $a, b \in \mathbb{N}$ show $\mathbb{Q}[\sqrt{a} + \sqrt{b}] = \mathbb{Q}[\sqrt{a}, \sqrt{b}]$.

8. Compute the Galois group of the splitting field of $x^4 - 4$ over $\mathbb{Q}$. Find all the intermediate subfields.