

HW 6 Comments

MATH 3360 Applicable Algebra

(Assignment due March 16, 2018)

Christine McMeekin

Friday March 23, 2018

Here are a few comments I had on HW 6. Note that these comments are not intended to serve as solutions. *Comments on how the problem was scored are in purple italics.*

Chapter 9

2. There is a short-cut; by Euler's theorem it is not actually necessary to check whether $[a]^4 = [1] \pmod{9}$ or whether $[a]^5 = [1] \pmod{9}$ because $\varphi(9) = 6$ and 4 and 5 do not divide 6.

I did not require that people know this for full points since Euler's theorem comes after these exercises.

8. One general strategy to show this is to show that k is common multiple of d and e if and only if $a^k \equiv 1 \pmod{m}$ then appeal to minimality.
29. Fermat's Little Theorem is useful here. *Always be sure to cite the results you use.*
43. Use Euler's Theorem (Theorem 6) and Proposition 7 noting that $322 = 20 \times 16 + 2$.

I only gave full credit here for computing the answer efficiently and otherwise a correct answer received a score of 2 out of 3.

60. One strategy was to use induction to get the result for prime powers then show that if the result holds for $n = a, b$ such that $(a, b) = 1$ then it also holds for the product.

Hint for a slick proof: Consider

$$\frac{1}{n}, \frac{2}{n}, \dots, \frac{n}{n}.$$

Reduce the fractions and group them by (reduced) denominator $d|n$.

This one was not graded for rigor.

Chapter 11

3. There are 5 such products.

We can't assume $abcd$ is well-defined in this question because that's what we're trying to show.

11. Working in U_{19} ,

$$\langle [7] \rangle = \{[1], [7], [11]\}$$

$$\langle [12] \rangle = \langle [8] \rangle = \{[1], [8], [7], [-1], [-8], [-7]\}$$

This answer is only unique up to choice of representatives mod 19.