We have a product of cyclic groups of prime power order. Let's call them

$$\mathbb{Z}_{2^{e_1^2}}, \mathbb{Z}_{2^{e_2^2}}, \mathbb{Z}_{2^{e_3^2}}, \ldots$$

$$\mathbb{Z}_{3^{e_1^3}}, \mathbb{Z}_{3^{e_2^3}}, \mathbb{Z}_{3^{e_3^3}}, \ldots$$

$$\mathbb{Z}_{5^{e_1^5}}, \mathbb{Z}_{5^{e_2^5}}, \mathbb{Z}_{5^{e_3^5}}, \ldots$$

$$\vdots$$

where for each p, the sequence of exponents $e_1^p \geq e_2^p \geq e_3^p \geq \ldots$ is eventually zero. (Here the superscript ^{p} is just to say who's who, it's not itself an exponent.)

Using CRT, put these together column by column:

$$\mathbb{Z}_{2^{e_i^2}} \times \mathbb{Z}_{3^{e_i^3}} \times \mathbb{Z}_{2^{e_i^5}} \times \cdots \qquad \cong \qquad \mathbb{Z}_{\prod_p p^{e_i^p}}$$

These will be our groups $\mathbb{Z}_{m_i}$, in reverse order. So to find out how many factors $\mathbb{Z}_{m_i}$ we need, let r be the maximum such that for some p, $e_r^p > 0$. That'll give the last $\mathbb{Z}_{\prod_p p^{e_i^p}}$ that isn't just $\mathbb{Z}_1$.

By the inequalities $e_1^p \geq e_2^p \geq e_3^p \geq \ldots$, it's obvious that each m_i divides m_{i+1}.

Try out that recipe with

$$\mathbb{Z}_{2^3}, \mathbb{Z}_{2^1}, \mathbb{Z}_{2^0}, \ldots$$

$$\mathbb{Z}_{3^7}, \mathbb{Z}_{3^0}, \ldots$$

$$\mathbb{Z}_{5^2}, \mathbb{Z}_{5^2}, \mathbb{Z}_{5^0}, \ldots$$

to convince yourself.