Distinguishing between the 3 kinds of properties–*pointwise*, *local*, and *global*–does not need to be difficult, but it can be at first glance. Some reasons for this confusion:

- A function f is *totally determined* by its value on each point of its domain. I.e., if we know the value of f on every point it is defined, we know all there is to know about f. (Does this mean that every property can be verified pointwise? It does not.)

- A property P that is a pointwise property can not merely be tested at a single point, but must be tested *at each point* of the domain, in order to hold that $P(f)$ is true.

- If we "know a function *globally*," then in particular we know what its value is on specific points.

The above statements seem to justify a confusion of pointwise vs. global (local falling ambiguously somewhere in the middle). If we're careful about how the 3 are defined and *a fortiori* how they are related, we will see precisely how to use them to classify, without ambiguity.

A property P is called **pointwise** if $P(f)$ can be verified by a nearly identical property $\overline{P}$ holding at each point x in the domain D_f. I say *nearly identical* because, e.g., "> 0" is used in 2 slightly different ways in $f > 0$, and $f(c) > 0$. (the second is a statement about a specific number, $f(c)$).

Equivalently, P is pointwise if in order to be false, one can find a single point x such that $\overline{P}(f(x))$ is false. E.g., "sin *is non-negative*" fails at the point $x = -\frac{\pi}{2}$, where $\sin(x) \not\geq 0$, hence "*is non-negative*" is a pointwise property of real functions.

Some properties are not pointwise properties!
E.g., f is differentiable if it is differentiable *at each point of its domain*, **TRUE!** But, being differentiable at x is really a statement about all sufficiently small neighborhoods around x. If I ask you "is your function, f, differentiable at 0?" and you reply "$f(0) = \sqrt{2}$" can I reach a conclusion?

No, I cannot.

Differentiability is a property that must be verified by looking at some sufficiently small neighborhood of each point x on the domain. In this case P, a statement about functions has an analog, $\overline{P}$, a statement about sufficiently small neighborhoods of some point, specifically differentiability at x. To fail it must fail on *every neighborhood* of some x.

Such a property is called **local**.

Some properties are not local properties!
E.g., that f attains its maximum is certainly not a pointwise property. Knowing the value of f *at* x does not tell us that $f(x) \geq f(y)$ for all y in the domain, nor does knowing arbitrarily small neighborhoods around each x in the domain. Certainly we can decide locally (at some point) if f has a local maximum, but that doesn't tell us how this local maximum compares to values of f evaluated far from x. Said differently, to claim that f **does not** attain its maximum on its domain we must supply the entire behavior of f, **all at once**, to substantiate the claim.

Such a property is called **global**.

Finally, if we wish to classify properties, **uniquely**, into these three sets, $\mathcal{P}_p$, $\mathcal{P}_\ell$, $\mathcal{P}_g$ (for, resp., pointwise, local, global), then we need to recognize:

$$\mathcal{P}_p \subset \mathcal{P}_\ell \subset \mathcal{P}_g.$$

(In the sense that, e.g., some properties **cannot** be verified pointwise, but that properties that can be verified pointwise may as well be verified locally or globally, etc.) But then adhere to the convention that "P is a local property" is understood as

$$P \in \mathcal{P}_\ell \quad \text{AND} \quad P \notin \mathcal{P}_p.$$

I.e., P can be verified locally, but cannot be verified pointwise.

In this way we classify each property P as uniquely one of the 3 given types of properties.