MATH 2310 QUIZ

Friday 2 October 2009. You have 50 minutes. No calculators are permitted.

(1) True or False? (If true, explain why. If false, give a counterexample.)

(a) No linear system has exactly two solutions.

(b) If A is *any* $n \times n$ matrix, then the matrix $I_n - AA^T$ is symmetric. (Here, I_n denotes the identity matrix of size $n \times n$.)

(2) Let

$$A = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}$$

Calculate each of the following. If it is not defined, say so.

(a) $A(C + B^T)$

(b) B^{-1}

(3) Let $X = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.

(a) Show that $X^2 - 7X + I_2 = 0$.

(b) Determine whether X is invertible, and find X^{-1} if it exists.

(4) Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 9 \\ 3 & 1 & 12 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) Find all solutions to the linear system $A\mathbf{x} = \mathbf{b}$.

(b) Is A invertible? Explain why or why not.

(5) Determine whether the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

is invertible, and find A^{-1} if it exists.

[END]