MATH 2930, Fall 2018
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Sections: 212, 217

Name:__________________________

Chapter 1.1 Review

Objectives: (1) To motivate the study of differential equations through an example. (2) To practice sketching direction fields. (3) To make qualitative statements about differential equations without solving them, i.e. finding equilibrium solutions, and long time behavior.

- Equations containing derivatives are called ____________________________.
- In order to study physical problems such as ____________________________, ____________________________, ____________________________ it is necessary to know something about differential equations (there are many, try to think of three).

Motivating Example: Suppose that an object is falling in the atmosphere near sea level. Formulate a differential equation that describes its motion (*Hint: Recall Newton's second law F=ma. What are the forces acting on the object as it falls?*).

Direction Fields: Consider the differential equation that you found in the previous example. For simplicity, lets assume that that $m = 10$kg (mass of object), $\gamma = 2$kg/s (drag force). Your differential equation should then become

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}. \tag{1}$$

Suppose that the velocity v has a certain given value. Then, by evaluating the right-hand side of equation (1), we can find the corresponding value $\frac{dv}{dt}$. For example, if $v = 40$, then $\frac{dv}{dt} = 1.8$. This means that the slope of a solution $v(t)$ has the value 1.8 at any time where $v = 40$. A sketch of the slopes for various values of v and t in the $tv-$plane is called a ______________________.

Sketching Direction Field: Sketch a direction field for equation (1).

A solution of a differential equation that does not change with time is called an ________________________.

Qualitative Analysis: Using your direction field as a guide, find an equilibrium solution to equation (1). Based off of your sketch, how does a solution to (1) behave as $t \to \infty$? What do you think is a physical interpretation of an equilibrium solution? (*Hint: In the context of this problem, the equilibrium solution is sometimes called the terminal velocity*).

Additional Excersies:

1. Sketch a direction field for $y' = 3\sin t + 1 + y$.

2. A population of insects in a region will grow at a rate that is proportional to their current population. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. Write down a differential equation that models the population at a given time. Assume that the proportionality constant for the birth rate is .2 and that there are initially 100 insects in the area. Sketch a direction field to try to determine whether the population will survive or not?