Chapter 1.1, 1.3, 2.1 Review

Objectives: (1) Identify dependent vs. independent variables in differential equations. (2) Draw direction fields (3) Recognize and solve 1st order linear ODEs with integrating factors

Part 1. For the differential equations below, do the following: state the dependent and independent variables, state the order of the equation, and determine whether it is linear or nonlinear.

1.
$$(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$$

2.
$$y'''' + 4y''' + 3y = x$$

$$3. \ \frac{d^2g}{dx^2} + \sin\left(x+g\right) = \sin x$$

4.
$$f' = f(f - 3)$$

Part 2. Sketch a direction field for the following differential equations.

1.
$$y' = y(y+3)$$

Part 3. State the general solution of the following general first order linear differential equation (Hint: integrating factor).

$$\frac{dy}{dt} + p(t)y = g(t)$$

Part 4. State whether each of the differential equation below are separable or not. Then, solve each one.

1.
$$y' + 3y = t + e^{-2t}$$

2.
$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, y(\pi) = 0, t > 0$$

3.
$$y' = \frac{x^2}{y}$$

4.
$$xy' = (1 - y^2)^{1/2}$$

$$5. \ \frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$$

Part 5. Find the value y_0 for which the solution of the initial value problem below remains finite as $t \to \infty$.

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$$y' - y = 1 + 3\sin t, y(0) = y_0.$$