

Chapter 2.2 and 2.3 Review

Objectives: (1) Identify and solve separable differential equations (2) Practice modeling with first-order differential equations

Part 1- Separable Differential Equations.

1. Solve the differential equation

$$\frac{dy}{dx} = \frac{-x}{y}.$$

2. Solve the initial value problem

$$y' = \frac{2 - e^x}{3 + 2y}, y(0) = 0$$

and determine where the solution attains its maximum value.

3. Solve the initial value problem

$$y' = \frac{1 + 3x^2}{3y^2 - 6y}, y(0) = 1$$

and determine the interval in which the solution is valid. Hint. To find the interval of definition, look for points where the integral curve has a vertical tangent.

Part 2- Modeling with First Order Differential Equations.

1. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

2. A young person with no initial capital invests k dollars per year at an annual rate of return r . Assume that investments are made continuously and that the return is compounded continuously.
 - (a) Determine the sum $S(t)$ accumulated at any time t .
 - (b) If $r = 7.5\%$, determine k so that \$1 million will be available for retirement in 40 years.

3. A tank contains 100 gal of water and 50 oz of salt. Water containing a salt concentration of

$$\frac{1}{4} \left(1 + \frac{1}{2} \sin t \right)$$

oz/gal flows into the tank at a rate of 2 gal/min, and the mixture in the tank flows out at the same rate. Find the amount of salt in the tank at any time.